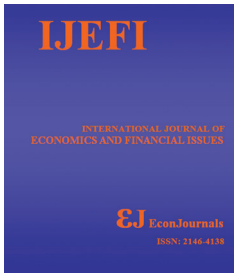

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The Effect of Public Investment on the Cycle and Economic Growth: A Simple Theoretical Model

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ABSTRACT

In the specialized literature, there are two versions of the effect of public investment on growth. One of them states that the relationship between them is positive; the other states that there is optimal public investment. This paper proposes an analytical theoretical scheme supporting that, in the short term, the relationship between public investment and economic growth is positive. However, in the long term, this relationship can still be positive or become negative depending on whether there is a crowding-out or a crowding-in effect. Whenever a crowding-in effect occurs, and the government does not have to decrease its investment, it does not increase debt or taxes, and the economy will be on a growth path. However, if there is a crowding-out effect, the trajectory of the economy will depend on the productive efficiency of the displaced capital.

Keywords: Public Investment, Economic Cycle, Economic Growth, Crowding-In, Crowding-Out
JEL Classifications: E10, E32, E60

1. INTRODUCTION

The economic crisis that began in the United States in 2008 has placed fiscal policy at the center of the academic and political debate. On the one hand, financial organizations, such as the International Monetary Fund, the European Central Bank, and the European Commission, have recommended to the European countries in crisis implementing fiscal austerity policies to regain the confidence of private investors and, with it, to encourage investment and growth. On the other hand, academics such as Krugman (2012) and Stiglitz (2010) have spoken out against austerity policies, arguing that they can exacerbate the crisis and have high social costs.

This debate is especially important when it focuses on public expenditures in investment because of the leading role that it plays in both the cycle and economic growth. There has been a lack of consensus on this issue because there are theoretical arguments both in favor and against increasing public investment. On the one hand, it is argued that public investment increases the productivity of private investment and thus generates an attraction effect on it (Baxter and King, 1993). On the other hand, it is claimed that public investment competes for resources with private investment to finance itself, by which it displaces private investment; in addition, excessive public investment can generate monopolies or distort assignments, causing negative effects on growth (Chen et al., 2017).

Based on the idea that public investment has as many positive as negative effects on economic growth, it has been contended that the relationship of this disparity on growth is non-linear. This idea has served as support to argue that there is a proportion of public investment in gross domestic product (GDP) that guarantees the highest GDP growth rate (Christie, 2014). At less than this proportion, it is possible to increase investment by obtaining higher GDP growth; however, if the investment is increased to greater than this proportion, economic growth decreases. In endogenous growth models, it is argued that the proportion of public investment in GDP that guarantees the highest growth rate depends on the productivity of public investment (Barro, 1990).

Barro (1990) argued that the proportion of public investment in GDP that maximizes growth is equivalent to the size public-product investment elasticity. Chen et al. (2017) contended that there is a proportion of public investment in total spending that guarantees maximum economic growth, and this proportion depends positively on the elasticity of the private investment-product between the sum of the elasticities of private investment and public investment with respect to the product. In these analytical schemes and as usual in the models of endogenous growth, full employment is assumed, as well as the offer of inelastic work and the exogenous population growth rate. Therefore, it is not possible to analyze the interaction between work and capital and to determine how changes in employment modify the results of fiscal policy. This limit has been overcome using dynamic general equilibrium models (Baxter and King, 1993); (Linnemann and Schabert, 2003) and (Leepper et al., 2010).

Baxter and King (1993) argued that changes in employment are provided by the "wealth effect," in other words, which is an increase in spending financed by a non-distorting tax, reducing the wealth of consumers and forcing them to consume less and work more. Subsequently, the increase in public investment causes the productivity of private investment grow, resulting in companies investing and producing more. The larger the public investment-product elasticity is, the greater the attraction effect will be of public investment on private investment.

Leepper et al. (2010) analyzed the effects of public investment on assignments and pricing when there are delays in implementing it. Similar to Baxter and King (1993), they showed that changes in employment are explained by the wealth effect. However, they argued that delays in private investment can cause companies to postpone their investment by waiting for public investment to occur and thereby gaining greater productivity from their investment. Moreover, they claimed that the greater that the delay in the implementation of public investment is, the lower that the effect is that it will have on private investment, but the more productive that the public investment is; in other words, the greater that the elasticity of the public investment-product is, the less important that the effect of the delay will be.

Linnemann and Schabert (2003) showed that an increase in public spending changes the level of employment through the wealth effect, similar to Baxter and King (1993), even in the presence of rigidities and an active monetary policy. Therefore, the wealth effect is present in the models of the New Neoclassical Synthesis (NNS) as the mechanism through which fiscal policy alters employment.

There are two aspects to highlight in the manner that public investment is usually postulated to modify prices and economic allocations.

1. In endogenous growth models, how fiscal policy modifies employment is usually not analyzed. In models of dynamic general equilibrium, it is often argued that an increase in public investment changes the level of employment through the "wealth effect"; therefore, it is the change in job offers that explains how employment varies. Thus, unemployment is always voluntary.
2. The larger the elasticity is of public investment-product (i.e., the more productive that public investment is), the greater the attraction is of the effect of public investment on private, or the greater the public investment/GDP ratio that maximizes growth will be.

The first point has caused great dissatisfaction with the in which economic theory explains how public investment modifies employment levels because, since the 2008 crisis, high unemployment rates have been observed despite the reduction in real wages. A clear example of the above is the case of the Spanish economy, in which, since the onset of the crisis real wages have been reduced by >25%; however, the unemployment rate is > 16%, so it is not plausible to argue voluntary unemployment as the theoretical model postulate. Coupled with the above, when there is a high unemployment rate, it is unlikely that public investment can increase employment by motivating consumers to increase job offers, as the wealth effect postulates.

For Hank and Solow (1995), the problem with current macroeconomic theory is that it has at its core a model -- the real cycle model -- that prevents the study of major economic pathologies, such as involuntary unemployment, which gave meaning to the birth of the macroeconomic theory. The first attempts to overcome these limitations are found in Velázquez (2015) and Velázquez and González (2016), and in these works public spending in consumption and public debt in general equilibrium scenarios are studied that do not have at their core the real cycle model; to achieve this goal, the analytical framework of the Theory of the Non-existence of the Labor Market (TNELM) has recurred.

The TNELM was initially suggested by Noriega (2001) and (2006). In this analytical framework and analogous to the NNS, agents are rational, but unlike in the latter, it is postulated that producers maximize the rate of profit. Because of this postulate, companies will demand work regardless of the real wage, so there is no price vector that guarantees full employment. Therefore, transactions are verified in both full employment and involuntary unemployment situations, and when the latter occurs, the economy is, on balance, restricted by demand, the latter being the usual situation of the economy. Thus, in this analytical framework, it is possible to analyze how public investment modifies employment levels in situations of involuntary unemployment.

In this work, in a similar manner to Velázquez (2015) and Velázquez and González (2016), a model of general equilibrium is developed within the analytical framework of the TNELM to analyze the effects of public investment on prices and assignments. The model shows that changes in employment are due to public investment being able to increase both the effective demand and the productive capacity, and as a result, employment varies to adjust production to the new effective demand. Conversely, and in a manner consistent with the literature, it is found that the effect of public investment on growth depends on how productive this process is. Furthermore, it is shown that, when public investment is increased, there will be a crowding-in effect (crowding-out effect) on private investment, provided that the first investment generates more (fewer) resources than those that demand financing themselves.

If there is crowding in, the economy will be on a path of growth if tax revenues grow sufficiently to sustain unchanged public spending without increasing debt. However, if there is a crowding-out effect, the economy will be on a decreasing path, provided that private investment is more productive than public investment. This article is organized into five sections: The first corresponds to the introduction; the second introduces the theoretical model; in the third, the economy is analyzed in its stationary state; in the fourth, the effect is analyzed of public investment on the growth in both the short and long terms; and finally, the conclusions of the investigation are offered.

2. THE MODEL

In a decentralized economy of private property, in which consumers and producers are price takers, there is the hypothesis that there is one producer and one consumer that represent the remainder. It is assumed that there is a good (y) that lasts two periods of productivity. The majority of models of general equilibrium state that all transactions and processes occur at the same time, so time elapses without history. Robinson (1980) criticized this posture by arguing that time is historic and irreversible, and events occur explicitly. In the analytical approach proposed herein, the hypotheses that time is historic and that events occur one after the other are restated; therefore, similar to Velázquez (2015) and Velázquez and Gonzalez (2016), it is assumed that events occur throughout a period of productivity integrated with a finite succession of moments.

2.1. Hypothesis about Time

A period of productivity is integrated with a finite succession of moments and is defined as a non-empty set that has the following characteristics:

$t \in [a_t, a_{t+1}]$; $[a_t, a_{t+1}] \in t$ *where* $|a_{t+1} - a_t| = \psi; \psi > 0$; $a_i \neq 0; t \cap t+1 = \{a_{t+1} \mid a_{t+1} \in ty\ a_{t+1} \in t+1\}$ *where* $at+j$ *for everything* $j \in R$, *is by definition a moment*

The hypothesis about time is illustrated in the following Table 1.

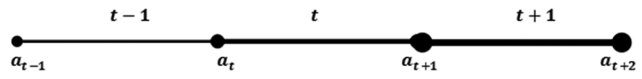
According to Table 1, a period can always be considered a positive distance between two moments.

2.2. Hypothesis about the Succession of Events

The following statements describe how events occur in period and will be valid for all periods.

Table 1:

Hypothesis about time



Where:
$t-1$ is drawn with the line
t is drawn with the line
$t+1$ is drawn with the line
Source: Velázquez (2015)

- At time a_t the productive period begins. Enterprises and government accrue debt to invest. The former indicates that investment decisions, both private and public, are made at the beginning of the period.
- Productivity starts at the moment $a_{t+\epsilon}$, where $a_{t+\epsilon} \in (a_t, a_{t+1})$. Therefore, it is after enterprises incur debt that they start to produce, and to do so, they demand labor work.
- At a_{t+1} the product is sold and bought; in addition, debts from the previous period and the taxes of the current period are paid.
- In moments when the end and the beginning of a period coincide, two interest rates coexist; one is a payment promise that determines the decision of investment; the other is the one that pays savings. In other words, at moment a_t are valid $(1+r_t)$ and $(1+r_{t+1})$.

In this research paper, it is assumed that agents form rational expectations but with two limitations: Agents are short-sighted, and expectations are not recursive.

2.3. Hypothesis about Short-Sightedness

Agents can anticipate events that are about to happen in their own periods, so they can form expectations about them, but they are not capable of seeing events that are about to occur in subsequent periods; thus, they cannot form expectations about these events, indicating that a person in moment a_t will form expectations of events that will occur at moment $a_{t+\varepsilon}$, if and only if $a_{t+\varepsilon} \in t$. The idea underlying this hypothesis is that the sooner that an event is about to happen, the easier that it is for people to predict its effects to form expectations regarding such events and to act accordingly. However, the further in the future that the event is, the more difficult that it is to form expectations and make decisions about them.

2.4. Non-Recursivity of Expectations

Expectations about the consequences of an event follow an order and are not recursive, indicating that, if agents expect that, at the moment that event z happens, the variable x is modified and alters y, then they are not capable of perceiving how the change in y will modify z or x. In this paper, it is assumed that expectations follow the order stated in the succession of events.

2.5. Consumers

The life horizon of consumers consists of two periods. In the first, they are young, and in the second, they are old. In each period, one young consumer coexists with one old consumer. Population does not grow because, when a consumer dies at the end of his/her second life period, another starting his/her first life period appears.

It is assumed that the labor supply of all consumers is inelastic (l_o), and this hypothesis is standard in the theory of growth but not in the real cycle theory.[1] To assume that the labor supply is inelastic has the advantage of guaranteeing that the conclusions from the model presented herein regarding aa phenomenon of great interest: Involuntary unemployment.

1 In the real cycle theory, the hypothesis that labor supply is elastic allows one to support that fluctuations in employment levels occur because of the rational behavior of consumers; therefore, unemployment is considered voluntary.

The optimizing conduct of the representative consumer is formalized through the following exercise of maximization.

$$Max U = c_{1t}^{\alpha} c_{2t+1}^{1-\alpha}$$

s.t $(1-\tau)[w_t l_o+\Pi_t]=c_{1t}+s_t$

$s_t (1+r_t)=c_{2t+1}$

with: $\alpha;\tau\in(0,1)$ (1)

In expression (1), c_{it+j} represents people's consumption, the subscript *i* indicates the life period in which the consumer is, the subscript *t+j* is in all of the variables and shows the period during which such a variable appears, τ is the income tax, which is considered constant, and *w* is the real wage. The labor supply does not have a temporary subscript because it is assumed to be inelastic and therefore constant in time, and Π is the enterprise income. It is assumed that young consumers are the owners of enterprises, and property rights are transferred without cost from generation to generation[2]; is the savings, and is the real interest rate.[3]

By solving the suggested maximization exercise in (1), the sale and purchase plans of the consumer are obtained:

$$c_{1t}=(1-\tau)\alpha[w_t l_o+\Pi_t] \quad (2)$$

$$c_{2t+1}=(1-\tau)(1+r_{t+1})(1-\alpha)[w_t l_o+\Pi_t] \quad (3)$$

$$s_t=(1-\tau)(1-\alpha)[w_t l_o+\Pi_t] \quad (4)$$

Equations (2), (3) and (4) are the standard results of the theory, and they simply show the income proportions allocated to consumption and to savings.

2.6. Producers

Similar to the works by Noriega (2001), Velázquez (2015), and Velázquez and González (2016), it is stated that the enterprise maximizes its income rate (π) and recognizes technology as a restriction. The optimizing conduct is formalized through the following maximization exercise:

$$Max(1+\pi_t)=\frac{y_t}{w_t l_t+(1+r_t)k_t}$$

$$y_t = A\left(l_t - l_t^*\right)^{\beta} k_t^{\gamma} g_{t-1}^{\delta}$$

Where $A,\beta,\gamma,\delta\in R^{+}$ $y\beta+\gamma+\delta<1$ (5)

Three elements about the formalization of optimizing the conduct of the enterprise are highlighted.

1. A rational enterprise is the one that maximizes its profitability rates. This statement diverges from the neoclassical hypothesis of enterprises having the objective to obtain maximum profit, which means the difference between the value of its sales and its costs.[4] It is important to mention that the idea of the profitability rate being the variable that guides an enterprise's decisions is not new; in classic and Marxist theories, enterprises guide their decisions seeking the maximum profitability rate. More recently, the finance theory of investment project evaluation emphasizes projects' profitability rates as part of the criteria that businesspeople consider in making investment decisions.
2. There are organization costs (l*). The hypothesis consists in part of the work that enterprises demand (l) being used to organize the production. For the price taker enterprise, these organization costs are constants; in other words, it is only a piece of information. However, such costs are the result of the market, which is why they are solved in general equilibrium. Noriega (2001) showed that the organization costs positively depend on the effective demand ($\hat{y}d$). The premise states that, as the demand that enterprises must fulfill grows, more organization is needed to address larger numbers of sale and purchase contracts.[5]
3. For the price taker enterprise, public investment (*g*) is a positive externality, indicating that, for businesspeople, the government's investment decisions are considered a piece of information because businesspeople are not capable of seeing that their own decisions can modify the government's behavior.

By solving the maximization exercise, the sale and purchase plans of the producer are obtained:

$$l_t = \frac{1-\gamma}{1-\beta-\gamma} l_t^* \quad (6)$$

$$k_t = \frac{\gamma}{1-\beta-\gamma}\frac{w_t}{(1+r_t)} l_t^* \quad (7)$$

$$y_t = A\frac{\beta^{\beta}\gamma^{\gamma}}{(1-\beta-\gamma)^{(\beta+\gamma)}}\left(\frac{w_t}{(1+r_t)}\right)^{\gamma} l_t^{*\beta+\gamma} g_{t-1}^{\delta} \quad (8)$$

Equations (6), (7) and (8), refer to labor demand, capital (*k*), and product supply, respectively. These results are similar to those obtained by Velázquez and González (2016) and Velázquez (2015), although, in these works, the effects of public expenditure

2 This assumption does not mean that agents are altruistic; it is used because it simplifies calculations and allows for the accumulation of enterprises' profits.

3 Note that there is no currency in the model; therefore, the real factor of interest is the price of the product.

4 Velázquez (2013) compared two price taker enterprises. One of them maximizes the profitability rate and the other the mass rate, and they both use the same technology. It is shown that, as long as production can be disaggregated into as many productive units as necessary, and both enterprises use the same amount of supplies, then the enterprise that maximizes the profitability rate will obtain both a higher profitability rate and greater mass profits than the other company.

5 Rodríguez (2005) showed that, every time a cubic polynomial function is used to represent the production function of an enterprise, one can disregard the organization costs and obtain the normal results of the non-existence of labor market theory (TIMT), as suggested by Noriega (2001). This time and to simplify calculations, the organization costs are used in the analysis.

in investment were not studied. Two elements of the sale and purchase plans of the producer will be highlighted.

1. Labor demand is inelastic of the real wage; its magnitude is determined by organization costs. Advancing results and, as shown by Noriega (2001) and Velázquez (2013), organization costs depend on effective demand; that is, the greater that the effective demand is, the higher that the organization costs are. In this manner, equation (6) shows that labor demand depends on effective demand through organization costs.
2. Public investment does not represent a direct cost for producers, so it does not intervene in labor and capital purchasing decisions about them, but it does potentiate production, consequently modifying supply; therefore, it is stated that product supply is determined by public investment.

2.7. Government

It is assumed that government only spends on productive investment. The decision of how much to spend is discretionary; therefore, there is no objective function for the government. However, it is enclosed by its budgetary restriction, which is formalized by the following equation:

$$b_{gt}+\tau[w_t l_t+\Pi_t]=g_t+b_{gt-1}(1+r_t) \quad (9)$$

The right side of equation (9) represents the government's income, which it obtains by incurring public debt (b_g) and by collecting income tax from young consumers $\tau[wl+\Pi]$. The left side shows the government's expenses in investment and the payment of old debt.

3. GENERAL BALANCE IN STEADY STATE

In this section, the general balance in the steady state is studied. Consequently, the subscripts that indicate temporality are not used. However, this omission does not indicate a modification to the hypothesis that time is a succession of moments during which events occur one after the other. The section starts from a fixed point to later analyze the dynamics of the economy, bearing in mind that this point is a particular case of the latter.

The general balance in the analytic framework of TNELM is a balance restricted by demand, which is why only when the planned demand is equal to the effective demand will there be full employment. In the specialized literature, the balances restricted by demand usually appear because prices adjust slowly, leading to transactions out of balance (Argandoña et al. 1996). In contrast, in TNELM, this balance restricted by demand originates because the interaction between supply and demand is not capable of determining a real wage that guarantees full employment; therefore, transactions are performed in the presence of involuntary unemployment, causing effective demand to be inferior to the planned demand, and the level of production adjusts to the first. To analyze this result, labor supply and demand are substituted in the following in equation:

$$\frac{1-\gamma}{1-\beta-\gamma}l^* - l_o \leq 0 \quad (10)$$

Inequation (10) is the "labor market" and shows that both labor buying and selling plans are independent of real wages; in fact, they do not depend on any price. Labor demand depends on the organization costs; however, these costs are not prices, so they do not have to adjust so that labor demand and supply match. From the previous paragraph, two important implications can be deduced: (1) It is not possible to determine a real wage that guarantees full employment; and (2) there can be full employment or involuntary unemployment, so inequation (10) is a weak inequality. Consequently, this inequality can be restated as follows:

$$\frac{1-\gamma}{1-\beta-\gamma}l^* - \phi l_o = 0 \quad \text{Where } \phi\in(0,1) \quad (11)$$

In expression (11), ϕ is the percentage of the labor supply that is employed and paid by enterprises. When it is 1, there will be full employment, and the lower it is, the more unemployment that there will be. Walras' law states that, if there is involuntary unemployment, then another market will have an excess of demand, which is why, within the neoclassic framework, it is not possible to include equilibrium with involuntary unemployment. However, because TNELM equilibrium is a balance restricted by demand, it can be interpreted as an imbalance in the neoclassic theory.

Equilibrium in TNELM is defined as a price and allocation vector that, given the salary, makes consumers' sales and purchase plans mutually compatible with the sales and purchase plans of producers. Compared to the neoclassic theory, in TNELM, equilibrium is defined by achievable plans, that is, sales and purchase plans that consumers can finance through non-wage income and with the part of their labor supply that becomes employed and is paid by enterprises (Velázquez 2015).

In this manner, the achievable plans of consumers are:

$$c_{1t}^r = (1-\tau)\alpha\left[w_t l_t + \Pi_t\right] \quad (12)$$

$$c_{2t+1}^r = (1-\tau)(1+r_{t+1})(1-\alpha)\left[w_t l_t + \Pi_t\right] \quad (13)$$

$$s_t^r = (1-\tau)(1-\alpha)\left[w_t l_t + \Pi_t\right] \quad (14)$$

The difference between achievable plans (equations 12, 13 and 14) and consumers' plans (equations 2, 3 and 4) is that the first ones depend on the wage income that they really obtain, that is, from labor demand, while the second ones are determined by the wage income that they expect to obtain, in other words, from labor supply.

Based on the idea of achievable purchase plans, effective demand is defined as the demand plans that economic agents can finance, indicating that effective demand is:

$$y_d^r = c_1^r + c_2^r + g + k \quad (15)$$

Effective demand only coincides with planned aggregate demand when there is full employment. It must be noted that, if there is

unemployment, financing for achievable plans will be limited and so will be for effective demand, which concerns demand rationing. As a result, these plans incorporate the hypothesis of dual decision stated by Clower (1965), which says that to be able to purchase, one must sell first. Thus, families' achievable plans depend on the work that they have managed to sell to enterprises.

As of the definition of balance in TNELM, the following equations of excessive demand and the modification to Walras' law are obtained:

$$c_1^r + c_2^r + g + k - y = 0 \tag{16}$$

$$k + b \quad - s \quad = \tag{17}$$

$$\left(c_{1t}^r + c_{2t}^r + g_t + k_t - y\right) + (1+r)\left(k + b_g - s^r\right) = 0 \tag{18}$$

Equations (16) and (17) are the goods markets in current and future periods, respectively; equation (18) is the modified Walras' law. It must be noted that, in this law, the labor market does not appear because of equilibrium; in this analytic framework, it is compatible with both full employment and involuntary unemployment. According to this law, if the future goods market is in balance, the current market will be too. Therefore, it is necessary to analyze the equilibrium of only one market.

By substituting the corresponding sale and purchase plans in equation (16) and the organization costs resulting from equation (11), the following is obtained:

$$\left[A\frac{\beta^{\beta}\gamma^{\gamma}}{(1-\gamma)^{\beta+\gamma}}\left(\frac{w}{1+r}\right)^{\gamma}(\phi l_o)^{\beta+\gamma} g^{\delta} - \left(\frac{\gamma}{1-\gamma}\right)w\phi l_o\right](1+r) \tag{19}$$
$$+\left(\frac{\gamma}{1+\gamma}\right)\left(\frac{w}{1+r}\right)\phi l_o + b_g - b_g(1+r) -$$
$$A\frac{\beta^{\beta}\gamma^{\gamma}}{(1-\gamma)^{\beta+\gamma}}\left(\frac{w}{1+r}\right)^{\gamma}(\phi l_o)^{\beta+\gamma} g^{\delta} = 0$$

In equation (19) the interest rate of steady-state equilibrium is solved. Note that, by definition, $(1+r_t) = \frac{p_t}{p_{t-1}}$, which is why in steady state, there is: $p_t = p_{t-1}$; consequently $(1+r)=1$. When this outcome occurs, equation (19) is an equality of any real wage and positive employment level, so neither real wages nor the employment level is determined in the markets. There are two important characteristics to highlight about equilibrium in this analytic framework.

1. Similar to Keynes (1936), it is the effective demand and not the notional demand that sends signals to the market; in other words, prices and allocations are adjusted to guarantee that supply is the same as the effective demand.
2. Markets determine neither the employment level nor the real wage. The labor market exists in appearance but not in substance. In other words, there is labor supply and demand, but their interaction is not sufficient to determine the vector of prices and allocations, that is, real wages and employment levels, which is why Noriega (2001) stated that, in TNELM, the labor market does not exist, and the confluence of labor suppliers and demanders is called the labor sector.

In Velázquez (2013) and (2015), it is argued that wage indetermination means that competitive economies are integrated by at least two institutions[6]: Markets and social conventions that determine real wages. Additionally, it is recognized that real wage determination in TNELM is part of the pending agenda of this theory; hence, in this research, this limitation is recognized, and it is assumed that real wages are exogenous and constant.

To analyze labor, it is necessary to observe how organization costs are established since labor demand depends on them (see equation 6). Note that equation (11) does not determine organization costs; it only shows that they can be compatible with both full employment and involuntary unemployment. To resolve organization costs, the function of enterprise production in a goods market is substituted ($\hat{y}_d - y_o = 0$), to obtain:

$$l^* = \left(\frac{1-\beta-\gamma}{\beta}\right)\left(\frac{\hat{y}_d}{Ak^{\gamma}g^{\delta}}\right)^{\frac{1}{\beta}} \tag{20}$$

By substituting the equation (20) in (6), the following is obtained:

$$l = \frac{1-\gamma}{\beta}\left(\frac{\hat{y}_d}{Ak^{\gamma}g^{\delta}}\right)^{\frac{1}{\beta}} \tag{21}$$

Equation (21) is labor demand; similar to Keynes's employment function (1936), it shows that there is a direct relation between the level of employment and effective demand. The reason for this relationship is that enterprises adjust their production to valid effective demand, so if the latter increases, enterprises will hire more labor to increase their production. In contrast, the inverse relationships between labor demand and private and public capital are due to gross substitutability between the factors that the production function presents because, when capital increases, less work is required to produce goods. However, both investments are part of effective demand; therefore, their effect on employment is more complex. Although equation (21) shows the employment determination, it is not possible to fix its level of steady state because it depends on effective demand, which is solved by the level of employment. Nevertheless, it is possible to analyze employment levels and their variations out of the steady state because of the hypothesis of temporality, which allows for revising the allocations in sequence.

The idea that real wages and employment levels are determined according to the logic of the labor market is not absolutely new; in fact, this idea is present in most of the economic theories that

6 Institutions are understood as the game rules are socially accepted.

do not recognize the competitive equilibrium theory as a solid explanation of how markets work. For example, in Marxist theory, the determination of real wages and employment is external to the logic of the labor market (Féliz and Neffa, 2006). Another example of this idea can be found in the theory suggested by Keynes (1936). In this regard, Pérez and Neffa (2006) stated that, for Keynes, there is no real labor market, indicating that neither wages nor employment is determined by the interaction between labor supply and demand.

A similar abandonment of the labor market as an analytic category to explain employment and salaries was observed by Kalecki (1956). For this author, employment responds to the effective demand that enterprises face. Regarding salaries, they are determined by the level of monopoly and the organization of the labor force; thus, real wage setting reflects the distributive conflict of income among capitalists and workers.[7] The post-Keynesians also do not recognize the labor market as a relevant analytic category to determine employment and real wages (Panigo, 2006).

In the following section, the way in which an increase in public expenses -- financed by debt -- modifies allocations and prices is analyzed. A change in the latter indicates that the government's income and expenditures will vary. This research paper examines the hypothesis that, in periods subsequent to expansionary fiscal policy, the government adjusts its expenditures to guarantee that public debt does not change.

4. ANALYSIS OF FISCAL POLICY

The analysis of the way in which public spending modifies allocations comes out of the steady state, indicating that it is assumed that the economy is in a steady state with unemployment and that an exogenous shock -- an increase in public spending -- changes this situation. The analysis starts with the following equations:

$$\left(\frac{\gamma}{1-\gamma}\right)\frac{w_{t+1}l_{t+1}}{(1+r_{t+1})}=(1-\tau)(1-\alpha)\left[\begin{array}{l}A\left(\frac{\beta}{1-\gamma}\right)^{\beta}E\left(l_t^{\beta}\right)k_t^{\gamma}g_{t-1}^{\delta}-\\ \frac{\gamma}{1-\gamma}w_tE\left(l_t\right)\end{array}\right]-b_{gt} \quad (22)$$

$$k_{t+1}=\left(\frac{\gamma}{1-\gamma}\right)\frac{w_{t+1}l_{t+1}}{(1+r_{t+1})} \quad (23)$$

$$\hat{y}_{dt}=(1-\tau)\alpha\left[A\left(\frac{\beta}{1-\gamma}\right)^{\beta}l_t^{\beta}k_t^{\gamma}g_{t-1}^{\delta}-\frac{\gamma}{1-\gamma}w_tl_t\right]+$$
$$(1+r_t)(1-\tau)(1-\alpha)\left[A\left(\frac{\beta}{1-\gamma}\right)^{\beta}l_{t-1}^{\beta}k_{t-1}^{\gamma}g_{t-2}^{\delta}-\frac{\gamma}{1-\gamma}w_{t-1}l_{t-1}\right]$$
$$+k_{t+1}+g_t \quad (24)$$

$$y_t=A\left(\frac{\beta}{1-\gamma}\right)^{\beta}l_t^{\beta}k_t^{\gamma}g_{t-1}^{\delta} \quad (25)$$

$$b_{gt}+\tau\left(A\left(\frac{\beta}{1-\gamma}\right)^{\beta}l_t^{\beta}k_t^{\gamma}g_{t-1}^{\delta}-\frac{\gamma}{1-\gamma}w_tl_t\right)=g_t+b_{gt-1}(1+r_t) \quad (26)$$

Equation (22) is the equality savings-investment. Equation (23) is the demand for investment. Equation (24) is the effective demand. Equation (25) is production, and equation (26) is government's budgetary restrictions at the end of the period.[8] At moment a_t, the economy is at a steady-state equilibrium with involuntary unemployment, and the government decides to incur more public debt to finance an increase in its investment expenditures. It is worth noting that, according to the hypothesis of event succession, at moment a_t the government incurs debts, but its spending comes at a subsequent moment within the same period. Therefore, agents expect that $db_{gt}=dE(g_t)$, and since the government acts according to $dE(g_t)\cong dg_t$, it makes no difference to discuss an increase in expenditures or in the government's debt.

At moment a_t the enterprises decide how much to invest based on the tax rate. The latter is fixed to guarantee the balance between the supply and demand of loanable funds, i.e., between private investment and private and public savings. Note that, at moment a_t, the level of employment in period t is not yet fixed; therefore, it appears as an expectation. Variables of period $t+1$ are also expectations; however, because of the hypothesis of short sightedness, the agents are not capable of analyzing the manner in which events occurring at a_t modify them. To simplify the statement, the operator of expectations will only appear in variables specific to the period under study, which is determined in subsequent moments. The change in governmental policy initially modifies the tax rate. To analyze how the latter changes in the presence of an increase in public spending financed by debt, the differential of the equation (22) is calculated, obtaining:

$$\frac{d(1+r_{t+1})}{db_{gt}}=$$
$$\frac{(1-\tau)(1-\alpha)\left(\beta E\left(\frac{y_t}{l_t}\right)-\frac{\gamma}{1-\gamma}w_t\right)\frac{dE(l_t)}{db_{gt}}-1}{-\left(\frac{\gamma}{1-\gamma}\right)\frac{w_{t+1}l_{t+1}}{(1+r_{t+1})^2}}\gtreqless 0 \quad (27)$$

Inequation (27) shows the change in the interest rate in the presence of an increase in public debt. In this research paper, it

7 Kalecki (1943 and 1956) goes even further to analyze the fixing of wages as part of a distributive conflict among capitalists and workers. Kalecki (1943) stated that the main limit that governments must reach to maintain full employment is that capitalists consider it a threat to the *status quo*, as it lessens their political power. Therefore, they resist policies oriented toward full employment. Consequently, the problem of full employment is that it is not only beyond the logic of the labor market, but it also exceeds the scope of economic science and is located in the political science sphere.

8 To obtain equations (23) and (25), substitute (6) in (7) in (5), respectively.

is assumed that $w_{t+j} < \frac{(1-\gamma)\beta}{\gamma} E\left(\frac{y_{t+j}}{l_{t+j}}\right)$ for everything j. This condition indicates that the wage is such that, when expecting an employment increase, it allows families to have higher incomes; in other words, the wage is such that: $\frac{dE(\Pi_t + w_t l_t)}{E(l_t)} > 0$ for any employment increase, which is why the sign of equation (27) essentially depends on the numerator. The first term shows the expected increase in savings because agents expect employment to increase as a response to higher public spending, indicating that $\frac{dE(l_t)}{db_{gt}} > 0$. The second term shows the increase in public debt due to higher government expenditures. Consequently, the tax rate will decrease (will increase) (will not change) as long as the savings expectation increases more than (less than) (by the same amount as) public debt.

Observe that, in the presence of an increase in public debt, the expectation of agents is that savings increase because they expect that the level of employment increases too. However, why do agents expect employment to increase as a consequence of higher debt? To answer this question, it is necessary to consider that debt increases are used to finance higher government expenditures as part of effective demand. Such an increase in public debt and therefore in government expenditures indicates that agents expect effective demand to grow. The foregoing is the result of obtaining the expectation from equation (24) and its differential with respect to the expectations of effective demand and public debt, obtaining:

$$\frac{dE(\hat{y}_t)}{db_{gt}} = 1 \tag{28}$$

Equation (28) shows that agents expect effective demand to grow by the same amount the as government's debt, which is only true because the government announced that such debt will be entirely used to increase its expenditures. With the expectation of higher effective demand, people expect that enterprises increase their production and will hire more workers. This result is obtained by applying the operator of expectations to equation (21) and differentiating it from the employment and the public debt, resulting in:

$$\frac{dE(l_t)}{db_{gt}} = \left(\frac{1-\gamma}{\beta}\right)\left(\frac{1}{Ak_t^{\frac{\gamma}{\beta}} g_{t-1}^{\frac{\delta}{\beta}}}\right)\frac{dE\left(\hat{y}_{dt}^{\frac{1}{\beta}}\right)}{dE(\hat{y}_{dt})}\frac{dE(\hat{y}_{dt})}{dE(g_t)} > 0 \tag{29}$$

The change in the interest rate determines the modifications in the investment decisions of enterprises. By performing the differentiation of the equation (23) and placing it in (27), the following is obtained:

$$\frac{dk_{t+1}}{db_{gt}} = (1-\tau)(1-\alpha)\left[\beta E\left(\frac{y_t}{l_t}\right) - \frac{\gamma}{1-\gamma} w_t\right]\frac{dE(l_t)}{db_{gt}} - 1 \gtrless 0 \tag{30}$$

Equation (30) shows how investment reacts to the government's decision to increase its expenditures. There are two forces that interact and explain how private investment will be modified. The first is the expectation of an increase of savings (first term of the inequation). The second is the increase in public debt (second term). Every time agents expect savings to grow more than (less than) (by the same amount as) the government's debt, investment will increase (decrease) (not change). The reason is that higher government expenditures compete for resources with private investment but also because it can generate more resources. Then, every time that public expenditure generates more (fewer) resources than it needs for its financing, private investment will increase (decrease). Arestis and Sawyer (2003) used similar reasoning to explain under which conditions public expenditure can lead to a crowding-out or crowding-in effect.

The change in investment determines the economy's behavior in both the short term and long term, which is why it is important to analyze it in detail. For the short-term analysis, there are two relevant scenarios: (1) when agents expect the employment level to grow as a consequence of higher government expenditures; and (2) when agents expect employment levels not to be modified.

In the first scenario, major public expenditures will cause a partial crowding-in or crowding-out effect on investment, that is, $\frac{dk_{t+1}}{db_{gt}} > -1$, due to the expectation that higher public expenditures cause agents to expect higher incomes, and consequently, people expect that at least part of the cost of increasing government expenditures is covered by the increase in such income.

In the second scenario, there is a total crowding-out effect, indicating that private investment is reduced by the same amount that government expenditures are augmented; therefore, $\frac{dk_{t+1}}{db_{gt}} = -1$. It is important to clarify that this scenario is an anomaly within this model because it will only occur when agents expect that major public spending does not modify effective demand, although the latter is a component of the demand *per se*.[9] However, given the relevance that has been granted to the total crowding-out effect in economic theory, it is important to analyze it.

4.1. Partial Crowding-in or Crowding-Out Effects over the Short Term

Every time that major government expenditures partially increase or decrease investment, the effective demand and

9 The Ricardian equivalence hypothesis states that major public spending does not increase the agents' income because they know that, sooner or later it, will be paid with higher taxes, so they reduce their consumption to increase their savings, and with the latter, they pay major future taxes. In this way, the Ricardian equivalence presumes that demand would not be modified due to the total crowding-out effect on public spending and private consumption.

employment will increase. To observe the aforementioned, the differential of equation (24) is calculated with respect to the increases in debt and effective demand. Assuming that $dE(g_t)=dg_t$, we obtain:

$$\frac{d\hat{y}_{dt}}{db_{gt}}=(1-\tau)(1-\alpha)\left[\beta E\left(\frac{y_t}{l_t}\right)-\frac{\gamma}{1-\gamma}w_t\right]\frac{dE(l_t)}{db_{gt}}>0 \qquad (31)$$

Inequality (31) shows the change in effective demand in the presence of a government expenditure increase. According to (31), every time major government expenditures cause a crowding-in or a crowding-out effect on private investment, demand will increase. The foregoing occurs because major government expenditures will increase private investment or, in a worst-case scenario, will reduce it to an amount less than the increase in public investment.

The change in effective demand modifies the level of employment because enterprises hire labor forces to adjust their production to the new effective demand. Based on equation (21), we obtain:

$$\frac{dl_t}{db_{gt}}=\frac{\frac{1}{\beta}\frac{l_t}{\hat{y}_{dt}}\frac{d\hat{y}_{dt}}{db_{gt}}}{1-\frac{1}{\beta}\frac{l_t}{\hat{y}_{dt}}\frac{d\hat{y}_{dt}}{dl_t}}>0 \qquad (32)$$

Inequality (32) shows the employment change before an increase in government expenditures. The denominator will be positive as long as $\beta>\frac{l_t}{\hat{y}_{dt}}\frac{d\hat{y}_{dt}}{dl_t}$, in other words, if the elasticity of the labor supply (β) is higher than the elasticity of labor demand $\left(\frac{l_t}{\hat{y}_{dt}}\frac{d\hat{y}_{dt}}{dl_t}\right)$. Velázquez (2015) stated that this condition is an efficiency condition of a productive system because it indicates that labor generates more resources than it consumes; in addition, it is shown that this condition is proved systematically. Throughout this research paper, it is assumed that this condition is confirmed for all periods. Consequently, the sign of inequality (32) is explained by the numerator; therefore, it is sufficient that effective demand has been augmented by an increase in government expenditure, so employment is too. In contrast, production is adjusted to effective demand, which is why it also increase.

4.2. Total Crowding-out Effects over the Short Term

This scenario is characterized because of the increase in government expenditures reduces investment by the same amount that it is augmented, which is only proved if agents expect that major government expenditures do not affect employment, in other words, $\frac{dE(l_t)}{db_{gt}}=0$. Under this condition, expressions (31) and (32) will equal zero because major government expenditures reduce investment by the same amount that it is increased, then and the effective demand does not change; thus, enterprises do not have incentives to modify their production and to continue hiring the same number of workers.[10]

4.2.1. Fiscal balance over the short term

Changes in employment levels and production modify the fiscal balance in the short term, and to see this outcome, it is sufficient to differentiate equation (26) with respect to public debt.

$$1+\tau\left(\beta\frac{y_t}{l_t}-\frac{\gamma}{1-\gamma}w_t\right)l_t'=\frac{dg_t}{db_{gt}}+b_{gt-1}\left(\frac{\gamma}{1-\gamma}\right)\frac{w_t}{k_t}l_t' \qquad (33)$$

On the left side of equation (33), there is the government's income from loans and the increase in the tax base; on the right side, there are major expenditures explained by the public expenditure increase and the cost of debt. Note that, based on equation (23), we obtain: $\frac{d(1+r_t)}{db_{gt}}=\left(\frac{\gamma}{1-\gamma}\right)\frac{w_t}{k_t}l_t'$. In other words, the cost of old debt increases because, by increasing employment, capital's profitability increases too.

Every time there is a total crowding-out effect, we will have that $l'_t=0$; therefore, $dg_t=db_{gt}$. Nevertheless, when there is a partial crowding-in or a crowding-out effect, it will be noted that $l'_t>0$. In this scenario, it will be assumed that $\tau\left(\beta\frac{y_t}{l_t}-\frac{\gamma}{1-\gamma}w_t\right)l_t'\cong b_{gt-1}\left(\frac{\gamma}{1-\gamma}\right)\frac{w_t}{k_t}l_t'$ such that $dg_t\cong db_{gt}$. In other words, it is supposed that, for all scenarios, public spending increases by the same amount that debt did.

These results are similar to those of Velázquez (2015) when the effect is analyzed of government's spending on consumption in the short term. Hence, the way in which government spending affects the short term does not depend on what the money is spent. In contrast, over the long term, it is fundamental to use spending to explain the path that the economy will follow.

4.2.2. The effect of public spending in the long term

The study of government's spending effect over the long term is divided into two scenarios:

1. When major government spending is attractive but not modified private investment; and
2. When major government spending totally or partially displaces private investment.

10 The fiscal multiplier is the other side of the crowding-in or crowding-out effects. In the short term, the multiplier will be $\frac{dy_t}{db_{gt}}=\beta y_t^{\beta-1}\frac{dl_t}{d_{bgt}}$. Note that, since production is adjusted to valid effective demand, there is: $\frac{dy_t}{db_{gt}}\approx(1-\tau)(1-\alpha)\left[\beta E\left(\frac{y_t}{l_t}\right)-\frac{\gamma}{1-\gamma}w_t\right]\frac{dE(l_t)}{db_{gt}}$. Consequently, in the presence of crowding-in effects (partial crowding-out effects) (total crowding-out effects), the multiplier will be greater than one (less than one, but positive) (zero).

4.2.3. Crowding-in effects in the long term

At moment a_{t+1}, agents expect savings to increase due to government's investment and expenditure growth in the previous period because investments -- private and public -- are components of the demand; however, for the period immediately following (period *t+1*, both are components of the economy's productive capacity. Therefore, agents expect income to increase, together with families' capacity to save money. This expected increase in savings causes interest rates to be reduced. To see this outcome, evaluate equation (22) in *t+1* and derive government's spending in *t* and interest rate in *t+2,* obtaining:

$$\frac{d\left(1+r_{t+2}\right)}{dbg_t}=\frac{(1-\tau)(1-\alpha)\left[\gamma E\left(\frac{y_t}{k_{t+1}}\right)\frac{dk_{t+1}}{db_{gt}}+\delta E\left(\frac{y_t}{g_t}\right)\right]}{-\left(\frac{\gamma}{1-\gamma}\right)\frac{w_{t+2}l_{t+2}}{\left(1+r_{t+2}\right)^2}}<0 \quad (34)$$

Equation (34) shows an interest rate decrease. Note that, in contrast with inequation (27), expectations about employment are not modified because a government spending increase occurred in the previous period; consequently, employment modifications are due to alterations in current allocations. Nevertheless, economic agents are so small that they do not notice how their decisions will modify the level of employment. However, all agents know how public and private investment changed in the previous period and include this information in their expectations.

Interest rate reductions encourage enterprises to increase their investment such that:

$$\frac{dk_{t+2}}{db_{gt}}=(1-\tau)(1-\alpha)\left[\gamma E\left(\frac{y_{t+1}}{k_{t+1}}\right)\frac{dk_{t+1}}{db_{gt}}+\delta E\left(\frac{y_{t+1}}{g_t}\right)\right]>0 \quad (35)$$

Equation (35) indicates that investment grows by the same amount as how much agents expect savings to increase. Major investment will cause effective demand to grow. However, it is not the only cause of growth. Based on equation (24), the following is obtained:[11]

$$\frac{d\hat{y}_{dt+1}}{db_{gt}}=\left((1-\tau)\alpha+\tau\right)\left[\gamma\frac{y_{t+1}}{k_{t+1}}\frac{dk_{t+1}}{db_{gt}}+\delta\frac{y_{t+1}}{g_t}\frac{dg_t}{db_{gt}}\right]+\frac{dk_{t+2}}{db_{gt}}>0 \quad (36)$$

Equation (36) shows that growth of effective demand occurs for three reasons: (1) Major consumption of young people; (2) an increase in the public spending or consumption of elderly people; and (3) an increase in current private investment. The first two occur because major private and public investments in the previous period expand current productive capacity and therefore increase youth income in this period, leading to higher youth consumption and higher tax collection. At first sight, it is not clear whether the latter is used to finance major spending or to pay major interest on old debt. However, even when using it for debt payment, it will mean higher income for government creditors (elderly consumers) who will consume more.

The increase in productive capacity, explained by the previously largest private and public investments, and the increase of effective demand will encourage enterprises to adjust their labor demands to guarantee that firms produce everything that the market demands. From equation (21) evaluated in *t+1*, the following is obtained:

$$\frac{dl_{t+1}}{db_{gt}}\gtrless 0 \text{ if and only if } \frac{d\hat{y}_{dt+1}}{db_{gt}}\frac{b_{gt}}{\hat{y}_{dt+1}}\gtrless\frac{dk_{t+1}}{db_{gt}}\frac{b_{gt}}{k_{t+1}}\frac{dy_{t+1}}{dk_{t+1}}\frac{k_{t+1}}{y_{t+1}}+\frac{dg_t}{db_{gt}}\frac{b_{gt}}{g_t}\frac{dy_{t+1}}{dg_t}\frac{g_t}{y_{t+1}} \quad (37)$$

Expression (37) shows that the level of employment will increase (will not change) (will decrease) as long as the elastic debt of the government in with respect to effective demand in *t+1*, is higher than (equal to) (lower than) the sum of the product of the elastic debt of the government in *t* with respect to the investment in *t+1*, times the elasticity of the latter with respect to the product in *t+1*, plus the product of elasticity debt in with respect to government's spending in times the elasticity of the latter with respect to the product in

Observe that the first term of the previous condition shows the increase in effective demand, and the second term indicates the increase in productive capacity, both as a consequence of the public spending augmentation in the previous period. Then, based on expression (37), one can conclude that employment increases (does not change) (decreases) as long as major government expenditures increase effective demand by a higher (the same) (a lower) amount than that which caused productive capacity to grow. The aforementioned explains why producers modify employment levels to adjust their production according to the current effective demand.

It is important to highlight two characteristics of this analysis: (1) The dual character of private and public investments, both of which are capable of expanding the effective demand and the productive capacity, which is why they determine the change in employment level; and (2) over the short term, the adjustment between supply and demand occurs through amounts –employment -- and not through prices.

The first characteristic is similar to what Harrod (1939) obtained. However, this author focused his study on investment dynamics, assuming that employment behaves the same way. Harrod (1939) stated that it is the dual character of investment that determines the economy's dynamics; in other words, if investment

11 To obtain (36), substitute equations (24) and (26) into (24) such that $\hat{y}_{dt+1}=(1-\tau)\alpha\left(A\left(\frac{\beta}{1-\gamma}\right)^{\beta}l_{t+1}^{\beta}k_{t+1}^{\gamma}g_t^{\delta}-\frac{\gamma}{1-\gamma}w_{t+1}l_{t+1}\right)+\left(\frac{\gamma}{1-\gamma}\right)w_{t+1}l_{t+1}+k_{t+2}+b_{gt+1}+\tau\left(A\left(\frac{\beta}{1-\gamma}\right)^{\beta}l_{t+1}^{\beta}k_{t+1}^{\gamma}g_t^{\delta}-\frac{\gamma}{1-\gamma}w_{t+1}l_{t+1}\right)$, and derive this one from public debt in t.

generates superior (inferior) effective demand, and productive capacity increase, then enterprises will increase (decrease) their investments, as well as employment.

The second characteristic is similar to that obtained by Kaleckian models: as long as it is assumed that there is excess installed capacity, an increase of demand will not cause prices, but production, to increase (Loaiza and Osmar, 2012). Nevertheless, unlike these approaches, in the model herein proposed, it is not necessary to assume excess installed capacity. The way to explain variations in the level of employment is similar to that of Velázquez and González (2016) and Velázquez (2015). However, they differed from the normal results of dynamic balance models suggested by the new classic synthesis and the NSN. In this regard, Baxter and King (1993) showed that, if there are flexible prices and wages, fiscal policy modifies employment levels through the wealth effect; in other words, consumers modify their work supply to compensate for the changes that fiscal policy causes to their wealth. Linnemann and Schabert (2003) stated that the wealth effect explains employment changes even in the presence of inflexibility and monopolies. Therefore, the main difference between the way in which one explains how fiscal policy modifies employment within the analytic framework of the NSN and the analytic framework herein presented is that, in the first, unemployment is voluntary, while in the second, it is involuntary.

The government modifies its spending to adjust the changes in income and expenditures without generating new debt or increasing taxes. Based on equation (26), the following is obtained:

$$\frac{dg_{t+1}}{dg_t}=\tau\left(\left(\beta\frac{y_{t+1}}{l_{t+1}}-\frac{\gamma}{1-\gamma}w_{t+1}\right)l'_{t+1}+\gamma\frac{y_{t+1}}{k_{t+1}}k'_{t+1}+\delta\frac{y_{t+1}}{g_t}\right)-\left(1+r_{t+1}\right)-\frac{d\left(1+r_{t+1}\right)}{dg_t}b_{gt}\gtrless 0 \quad (38)$$

The first term on the right side of equation (38) shows the increase in the tax base due to the increase in consumers' income; the second term notes the increased cost of old debt ($1+r_{t+1}$); and the third term indicates the change in cost of debt, i.e., the interest rate. Generally, every time that tax income increases more than the total cost of debt, public spending will increase with no need to increase taxes or public debt. With the purpose of analyzing the necessary conditions for this outcome to occur, in this paper, how the total cost of public debt varies is studied. Note that the cost in which debt changes is $\left(1+r_{t+1}\right)+\frac{d\left(1+r_{t+1}\right)}{dg_t}b_{gt}$. In this research, it is considered that the first element is insignificant to explain the variation in debt,[12] and for this reason, it was only analyzed for the second element.

The change in the cost of debt depends on how interest rates have varied. In this scenario, interest rate $\left(1+\mathrm{r}_{t+1}\right)$ decreases in the immediate past period (t). However, in this period ($t+1$), employment changes modify it again. Based on equation (23), the following is obtained:

$$\frac{d\left(1+r_{t+1}\right)}{dg_t}=\left(\frac{\gamma}{1-\gamma}\right)\frac{w_{t+1}}{k_{t+1}}l'_{t+1}-\left(\frac{\gamma}{1-\gamma}\right)\frac{w_{t+1}l_{t+1}}{k^2_{t+1}}k'_{t+1}\geq 0 \quad (39)$$

The first term of equation (39) shows that, if employment increases, then the interest rate will increase; the second term is equivalent to equation (27), indicating:

$$-\left(\frac{\gamma}{1-\gamma}\right)\frac{w_{t+1}l_{t+1}}{k^2_{t+1}}k'_{t+1}=\frac{(1-\tau)(1-\alpha)\left(\beta E\left(\frac{y_t}{l_t}\right)-\frac{\gamma}{1-\gamma}w_t\right)\frac{dE\left(l_t\right)}{db_{gt}}-1}{-\left(\frac{\gamma}{1-\gamma}\right)\frac{w_{t+1}l_{t+1}}{\left(1+r_{t+1}\right)^2}}$$

,so only the reduction that initially suffered due to the interest rate in the last period is reflected. It is not possible to know a priori which effect will dominate. However, one can offer the analysis conditions:

a. If $\frac{dl_{t+1}}{db_{gt}}\leq 0$ then $\frac{dg_{t+1}}{dg_t}>0$

b. If $\frac{dl_{t+1}}{db_{gt}}>0$ and $\frac{l'_{t+1}}{l_{t+1}}\leq\frac{k'_{t+1}}{k_{t+1}}$ then $\frac{dg_{t+1}}{dg_t}>0$

c. If $\frac{dl_{t+1}}{db_{gt}}>0;\frac{l_{t+1}}{l_{t+1}}>\frac{k_{t+1}}{k_{t+1}}$ and $\tau\left(\left(\beta\frac{y_{t+1}}{l_{t+1}}-\frac{\gamma}{1-\gamma}w_{t+1}\right)l'_{t+1}+\gamma\frac{y_{t+1}}{k_{t+1}}k'_{t+1}+\delta\frac{y_{t+1}}{g_t}\right)>\left(1+r_{t+1}\right)+\frac{d\left(1+r_{t+1}\right)}{dg_t}b_{gt}$;

then $\frac{dg_{t+1}}{dg_t}>0$

Conditions (a) and (b) are sufficient but not necessary for public spending to be self-financing. In all of the conditions, debt does not increase; therefore, major fiscal income guarantees higher public spending. Condition (c) is a sufficient and necessary condition when the interest rate increases, and it only shows that tax income must increase more than the cost of debt so that expenditures can be self-financing. It is important to highlight two aspects of this condition. (1) The greater the elasticity of debt-employment is with respect to elasticity private debt-investment, the higher the interest rate will be. The aforementioned indicates that, when the increase in public spending is more successful in generating employment, it is more difficult for it to be self-financing. (2) The larger the public debt is, the more complicated

12 This insignificance can be noticeable if it is considered that the interest rate hardly surpasses 20%, so ; in contrast, public debt can increase to millions of dollars, so even when multiplied by the interest variation, it will be considerably higher.

it is for spending to continue increasing without increasing taxes or incurring more debt.

Conditions (a), (b) and (c) indicate that the "free lunch" hypothesis is not verified. This hypothesis states that an increase in public spending financed with debt sooner or later must be accompanied by major taxes to finance it. This theory is supported by the idea that national income, as well as the government's tax collections, is constant. However, in this analytic framework, it has been shown that, if there is a crowding-in effect, public spending can cause the economy to grow and therefore generate the income that it needs to be self-financing.

For the subsequent periods, the logic in which allocations and prices are modified is similar that of this period. Thus, if any of the 3 conditions occurs, then the economy will go on a path of steady growth. However, what happens if none of the 3 conditions occurs? In other words, what happens if public spending is not self-financing and must turn to a cutback to not increase debt or taxes? This question has an easy answer after analyzing the crowding-out effect in the long term, which is why its analysis is postponed until a later study.

4.2.3. Crowding-out effects in the long term

At moment a_{t+1}, agents are deciding how much to invest based on the interest rate. However, this rate varies because the previous private investment was reduced, but public investment increased in such a way that, based on (22) and similar to (34), the following is obtained:

$$\frac{d\left(1+r_{t+2}\right)}{dbg_t}=\frac{(1-\tau)(1-\alpha)\left[\gamma E\left(\frac{y_t}{k_{t+1}}\right)\frac{dk_{t+1}}{db_{gt}}+\delta E\left(\frac{y_t}{g_t}\right)\right]}{-\left(\frac{\gamma}{1-\gamma}\right)\frac{w_{t+2}l_{t+2}}{\left(1+r_{t+2}\right)^2}}\leq 0 \quad (40)$$

From expression (40) the necessary and sufficient conditions are obtained for the interest rate to increase, decrease or remain the same:

$$\text{If then } \gamma\frac{b_{gt}}{k_{t+1}}\frac{dk_{t+1}}{db_{gt}}\geq\delta\frac{b_{gt}}{g_t}\frac{dg_t}{db_{gt}};\text{then}\frac{d\left(1+r_{t+2}\right)}{dbg_t}\geq 0 \quad (41)$$

In equation (41), γ is the elasticity capital-product, and is the elasticity public expenditure-product. Expression (41) shows that, if the product of the elasticity capital-product times elasticity debt-capital is higher than (lower than) (equal to) the elasticity public spending-product times the elasticity debt-public spending, then the interest rate will increase (decrease) (not change). In other words, if private investment is more (less) (equally) productive than public investment, then the reduction of the first destroys more (fewer) (the same) resources than the increase of the second generates; consequently the interest rate will increase (decrease) (not change). These three scenarios will mark the path to be followed for the subsequent periods.

Note that, if the crowding-out effect was partial, then $\left|\frac{dk_{t+1}}{db_{gt}}\right|<\frac{dg_t}{db_{gt}}$, and if it was total, it will be $\left|\frac{dk_{t+1}}{db_{gt}}\right|=\frac{dg_t}{db_{gt}}$. Despite the previous determining how the interest rate varies, it does not modify the analysis of equation (41), which is why the situations resulting from this equation will be studied, regardless of whether the crowding-out effect was partial or total.

Situation (a) $\left|\gamma\frac{b_{gt}}{k_{t+1}}\frac{dk_{t+1}}{db_{gt}}\right|=\delta\frac{b_{gt}}{g_t}\frac{dg_t}{db_{gt}}$

In this situation, private investment is as productive as public investment; consequently, reducing private investment destroys as many resources as major public investment generates, which is why families expect savings to remain the same, so the interest rate is not modified. By keeping the interest rate constant, enterprises are not encouraged to change their investments; therefore, the latter does not change either. Effective demand is the same as that in the previous immediate period because both the current investment and the productive capacity of the economy did not change, which is why enterprises do not have incentives to modify their labor demands. As a consequence of the aforementioned, the economy goes through a new steady state characterized by higher levels of production, investment, and employment.

Situation (b) $\left|\gamma\frac{b_{gt}}{k_{t+1}}\frac{dk_{t+1}}{db_{gt}}\right|<\delta\frac{b_{gt}}{g_t}\frac{dg_t}{db_{gt}}$

In this scenario, private investment is less productive than public investment. Then, major public investment generates more resources than the fall of private investment destroys. Therefore, agents expect their income to grow, together with their savings. The expectation of higher savings will cause interest rate to decrease. As of this moment, this scenario is similar to the crowding-in effect.

Situation (c) $\left|\gamma\frac{b_{gt}}{k_{t+1}}\frac{dk_{t+1}}{db_{gt}}\right|>\delta\frac{b_{gt}}{g_t}\frac{dg_t}{db_{gt}}$

In this case, private investment is more productive than public investment, indicating that a decrease of private investment destroys more resources than those generated by major private investment; for this reason, families expect their income to decrease, together with their savings. The expectation of lower savings causes interest rates to increase, which will motivate enterprises to decrease their investments. In this manner, similar to equation (35):

$$\frac{dk_{t+2}}{db_{gt}}=(1-\tau)(1-\alpha)\left[\gamma E\left(\frac{y_{t+1}}{k_{t+1}}\right)\frac{dk_{t+1}}{db_{gt}}+\delta E\left(\frac{y_{t+1}}{g_t}\right)\right]<0 \quad (42)$$

The reduction of private investment in both the current and immediately previous periods will cause effective demand to decrease. Similar to (36), the following is obtained:

$$\frac{d\hat{y}_{dt+1}}{db_{gt}} = \left((1-\tau)\alpha+\tau\right)\left[\gamma\frac{y_{t+1}}{k_{t+1}}\frac{dk_{t+1}}{db_{gt}}+\delta\frac{y_{t+1}}{g_t}\frac{dg_t}{db_{gt}}\right]+\frac{dk_{t+2}}{db_{gt}}<0 \quad (43)$$

A fall in investment and productive capacity encourages enterprises to adjust their labor demands. Thus, expression (37) and its explanation remain valid for this scenario, except that the elasticity of public debt in -effective demand in *t+1* and the elasticities of the debt of government in *t*-investment in *t+1* are negative, indicating that both effective demand and productive capacity are reduced, so employment will increase (decrease or not change), as long as productive capacity falls by a higher (lower or equal) amount than effective demand does.

The government's budgetary restriction will be affected because, by decreasing families' incomes, the tax base is reduced. In contrast, the change in employment and the fall in investment modify the cost of old debt. Then, expression (38) is proved in this scenario, and its analysis is similar to the statement in the crowding-in effect study, as long as it is considered that the tax base decreases and that the fall of the previous investment increases interest rates, and major unemployment reduces them. Consequently, public spending will be reduced as a response to the fall of fiscal income and the increase in the cost of debt. This scenario will be reproduced for the remaining periods; therefore, it is going to be a downshifting scenario.

Returning to the question raised at the end of the study of crowding-in effect, what happens if public spending must be reduced to increase taxes or public debt? If this outcome happens, it will be a similar scenario to that studied in the long-term crowding-out effect with a significant difference, which is that it is the private investment that displaces public investment. Then, similar to equation (41), we obtain:

$$\text{If } \gamma\frac{b_{gt}}{k_{t+1}}\frac{dk_{t+1}}{db_{gt}} \le \left|\delta\frac{b_{gt}}{g_t}\frac{dg_t}{db_{gt}}\right| \text{ then } \frac{d\left(1+r_{t+2}\right)}{dbg_t} \ge 0 \quad (44)$$

The analysis of inequation (44) is similar that of inequation (41), indicating that, if in the first period, major public investment causes to private investment grow, but in the second period, insufficient fiscal income is generated to maintain the public investment growth; therefore, the latter must be reduced, and then if public investment is more (less) productive than private investment, the economy will be in a downshifting (growth) phase. Nevertheless, if both are equally productive, the economy will be in a new steady state.

5. CONCLUSIONS

The way in which public spending increases for investment -- which is financed with public debt -- affect growth has many aspects. In the short term, major public investment will increase employment and production. However, as long as public investment generates more (less) (the same) resources as it demands to become financed, then private investment will increase (decrease) (not change).

The behavior of both private and public investments will determine the growth path that the economy will follow in subsequent periods. If public investment generated a crowding-in effect on private investment, then as long as the sufficient fiscal resources are generated for public investment and are not reduced in subsequent periods, the economy will be situated on a growth path. Nevertheless, if the debt increase is more than the increase in fiscal income and the government reduces its investment to avoid incurring more debt or raising taxes, then the growth path is similar to that obtained when public investment displaces private investment, with the peculiarity that, in this scenario, it is private investment that displaces public investment.

Along the growth path, there are two factors that favor that the increase in the cost of public debt, compelling the government to reduce its investment so that it does not incur more debt or raise taxes: (1) the debt amount; and (2) employment growth. The aforementioned indicates that the more successful that the government is in increasing employment, the more difficult that it will be to reduce its expenditures.

In the presence of a crowding-out effect, in which public investment displaces private investment or vice versa, as long as the displaced investment is less (more) (equally) effective to that which was increased, the economy will be on a growth (downshifting) path. The previously mentioned points make not only how much the government spends on investments but what it spends on -- in other words, the efficiency of its spending -- extremely important

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