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A Mathematical Model for the Study of the Effects of the Economic Cycle on the Real GDP Growth Rate through the Expectations-Adjusted Phillips Curve

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ABSTRACT

In this paper is presented a theoretical model finalized to explain the effects of the economic cycle on the real GDP growth rate of a given country's economy towards a selected business partner. In other words, the present paper expose an innovative theoretical model, based on the expectations-adjusted Phillips Curve and on the Okun's law, which proposes the existence of a relationship between the difference in the effective inflation rate and in the expected inflation rate of the national economy of a generic country (inflation gap), and the growth rate of the real GDP of the national economy and of its corresponding commercial partner, both determined by the different phases of the economic cycle. The two economies examined, for the purpose of empirical verification, are Germany and the group of the countries belonging to the euro area, considered, as a whole, as a single economy. The under exam sample period ranges from the first quarter of 1999 to the first quarter of 2018. The result to which the authors arrive, appropriately verified with econometric models, indicates not only the existence, the significance and the robustness of the relationship established by it but also the ability of the model to predict, with good precision, the effects of the economic cycle on the real GDP growth rate, given the inflation gap.

Keywords: Expectations-adjusted Phillips Curve, Okun's Law, Business Cycle, Mathematical Models and Methods
JEL Classifications: C02, C32, E32, E37

1. INTRODUCTION

The international economy offers numerous theories to explain the effects of some variables, such as the nominal exchange rates, the interest rates and the inflation rates, on the competitiveness of a certain economy towards its partners. Most of these theories has been proposed in the literature during the XX century and has found wide diffusion in the academy thanks to their ability to explain rather exhaustively many phenomena of the economic reality for which adequate interpretations did not previously exist. Today these continue to be object of criticism and of research activity in order to perfect and to update them to the structural changes that have interested the economic systems during the time.

One of the main variables that influences the competitiveness of a certain economy, compared to the others, is the variation of the general price level, and that is, the inflation rate. Among the macroeconomic relations that involve the rate of inflation, the one of greatest interest for the academic community and for the political decision-makers is certainly the relationship, universally known as the "Phillips Curve," which, exposed for the first time by the New Zealander economist Phillips (1958), links the nominal salaries to the unemployment rate.

The Phillips's studies were taken up by Samuelson and Solow (1960), who, studying the period between 1934 and 1958 for the United States of America, explicated an inverse relationship between the inflation rate and the unemployment rate: for the

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first time, it was began to believe that the expansive monetary and the fiscal policies could be a useful instrument of economic policy to increase the inflation and to obtain a reduction of the unemployment rate.

The original formulation of the Phillips Curve began, subsequently, to be challenged in front of the problem of the stagflation which affected the major western economies in those years; in 1967-1968, in fact, Phelps (1967, 1968) and Friedman (1968) detected a series of misspecifications in Phillips's original relation and proposed some corrections to them.

The main of these corrections was the introduction of the inflation expectations, expectations that they assumed to be adaptive, that is, dependent on the inflation recorded by the operators in the previous periods; the introduction of these expectations led to the Phillips Curve in its modern formulation. A second and important correction was the ascertainment that the trade-off between the inflation gap and the unemployment gap was valid only in the short-medium term and that instead in the long run the Phillips curve was reduced to a vertical line.

In the second half of the 70s, Lucas (1976) and Lucas and Sargent (1979) introduced the concept of rational expectations, according to which the economic agents formulate their expectations no longer only on the basis of historical inflation values, but on the basis of all the information they have; in particular, the two economists theorized that an important role in this sense was covered by the announcements of monetary policy and by their credibility. The main conclusion that Lucas and Sargent reached, with their formulation of the curve, was that, in the short run, the monetary policies cannot influence the level of the aggregate production. Always starting from the second half of the 70s, the Keynesian economists began to develop the theory of the nominal rigidity, beginning the new Keynesian macroeconomics. In particular, Fischer (1977) and Taylor (1980) argued in their papers that prices and wages are not instantly adjusted to changes in the economic policy, because they are set in the long run. Therefore, regardless of the credibility of the economic policy announcements, prices and wages can not instantly incorporate the changes into the expectations. Also according to Fischer and Taylor, the adjustment of the expectations happens in the long term also because of the phenomenon of the "staggering wage agreements," for which the conclusion of the new contracts and the renewal of the old ones does not occur at the same time moment, but in different periods.

Since the early 90s, the research on the Phillips curve has intensified considerably in fact, numerous its alternative forms have been presented. In particular, the attention of the scholars has focused on the model's ability to predict the future inflation in a certain economy. Interesting in this regard are the papers proposed by Stock and Watson (2008), Atkenson and Ohnanian (2001) and Rondina (2018).

The main aim of this paper is to present a theoretical model, based on the expectations-adjusted Phillips Curve and on the Okun's Law, in which, given a country and its generic business partner, the difference between the effective inflation and the expected inflation in a given country is related to the effects of the economic cycle on the GDP of the national economy and of the economy of the considered partner.

In this model, finalized not only to explain the effects of the economic cycle on the GDP growth rate but also useful for forecasting purposes, the authors present a linear relationship that links the difference between the effective inflation rate and the expected inflation rate (inflation gap) of the national economy, with the difference between the cosine of the real GDP growth rate of the national economy and the cosine of the growth rate of the real GDP of the trading partner. This last difference represents the effect, on the inflation gap, of the economic cycle in the national economy and in the economy of the considered commercial partner; the inflation gap, by effect of the economic cycle in the two economies involved, follows, as the authors will later highlight, the trend of a periodic wave. A positive or a negative change in the inflation can lead to a positive or a negative change in the difference in growth between the national economy and the foreign economy, due to the effects of the economic cycle.

The model was obtained using the simplest formulations of Friedman's expectations-augmented Phillips Curve and of the Okun's law and assuming that the real exports and the real imports of Germany are a linear function of the real GDP (national and foreign), of the real exchange rate (RER), of the final consumption expenditure in real terms of the private operators and of the government (in the national and foreign economy), of the export price index, of the import price index and of the sine of the real growth rate of the national GDP and of the trading partner GDP. It has also been assumed (and verified) that the expected inflation at time $t+1$ is equal to the cyclical component of the effective inflation rate at time t, namely, that the operators formulate their expectations giving greater weight to the goods characterized by greater price volatility.

In the authors' opinion, this simple theoretical model, appropriately verified empirically, represents a useful predictive tool for the effects of the economic cycle on the real GDP of the national economy and of the economy of the business partner considered. In other words, the results, obtained from the econometric verification, demonstrate not only the existence and the significance of the relationship established by the theoretical model and the validity of the macroeconomic assumptions on which it is built, but also the ability of the latter to predict with good precision the effect of the economic cycle on the real GDP growth rate in the countries of the euro area, given the inflation gap in Germany.

The econometric analyses, carried out in the paper, also seem to suggest that economic operators, in a given economy, can formulate their own expectations about future inflation using the cyclical component of the effective inflation rate by Consumer Price Index, namely the short-term changes due mainly to goods that, according to economic theory (Taghizadeh-Hesary et al. 2018), traditionally have a greater price volatility, such as the electricity and the foodstuffs.

The model used appears statistically robust and internally valid, besides to present an innovative character; in fact, the topic

treated by the authors, in this article, seems not to have been the object of study by other authors until now; that is, currently there are no studies that relate the effects of the economic cycle to the expected inflation rate. Furthermore, the ability of the inflation, within a given economy, to determine and to predict the effects of the economic cycle on the real GDP of the national economy and of its trading partners constitutes undoubtedly an interesting contribution even with respect to the theory of international trade.

The paper is organized as follows.

In section 1, after the introduction, are analyzed the research aims, while in section 2 the instructions and procedures used are exposed; in section 3, instead, the theoretical model and the relative estimation methods are introduced. In section 4 the results of the model estimation are reported and some diagnostic tests related to them are examined; subsequently, it is carried out, through an ARMAX model, a static forecast of the GDP growth rate. Section 5, finally, provides a conclusion and contains a general discussion on the directions for future research.

The authors hope, furthermore, that the paper is a very useful for stimulating research at the interface of mathematics and economics (useful, namely, in bridging economics and mathematics, in particular) and that it builds a link that can become advantageous for both disciplines involved.

2. INSTRUCTIONS AND ESTIMATION PROCEDURES USED

In addition to what is specified in the research objectives, the authors add the following information on the instructions and procedures used in the research.

The estimation of the model parameters and their validation were carried out with historical series data on a quarterly basis and with the following type models: multivariate linear regression (OLS) and Autoregressive Moving Average with exogenous inputs (ARMAX). In particular, the procedure adopted in the paper consists in using the vector regression models (VAR) by means of which the authors have tried to expose an econometric analysis to study the effects of the inflation gap on the GDP growth rate, due to business cycle effects.

The economies examined for econometric verification are Germany and the euro area countries, considered, as whole, like a single economy. The sample period of the data ranges from the first quarter of 1999 to the first quarter of 2018.

The historical series relating to the final consumption expenditure of the private operators and of the government in Germany, to the final consumption expenditure of private operators and of the government in the euro area, to the German export price index and to the German import price index, were collected through the software Datastream of Thomson Reuters. All other variables come from the online database of the Federal Reserve Bank of Saint Louis (FRED). The two variables *hpt_GrowthEur* and *hpt_GrowthGer* (in the models 1 and 2 of the paragraph 4) were obtained applying a Hodrick-Prescott filter with $\lambda = 1600$, respectively to the real GDP growth rate of the euro area and to the real GDP growth rate of Germany. The variables *RERGer* and *REREur* have been computed by the authors using the nominal exchange rate, listed certain to uncertain, between Germany and the euro area (which, of course, is fixed) and the Consumer Price Index of both economies.

In the model 3 (of the paragraph 4) the exogenous variable *CyclicalInfGert1* was obtained applying a Hodrick-Prescott filter with $\lambda = 1600$ to the effective German inflation rate (calculated at the German CPI) and was modeled according to an AR(1) process.

3. THE THEORETICAL MODEL

Consider the relation expressed by Okun's law (Okun, 1962):

$$u_t - u_{t-1} = -\beta g_{Y_t} \tag{1}$$

where:
u_t is the unemployment rate at time t,
u_{t-1} is the unemployment rate at time $t-1$,
β is the Okun's coefficient,
g_{Y_t} is the effective growth rate of the production (or of the national income).

Since Okun's law, from an intuitive point of view, expresses the variation in income corresponding to a reduction in the effective unemployment rate, compared to the natural unemployment rate, the previous relation can be rewritten as:

$$u_t - u_n = -\beta g_{Y_t} \tag{2}$$

Consider also the relation expressed by the expectations-adjusted Phillips's Curve:

$$\pi_t - \vartheta \pi_{t-1} = -\alpha \left(u_t - u_n \right) \tag{3}$$

where:
π_t is the inflation rate at time t,
π_{t-1} represents the adaptive inflation expectations at time $t-1$,
ϑ end α are constants between 0 and 1.

In the case in question it is assumed that the expected inflation is exactly equal to the cyclical component of the effective inflation registered at time t, that is:

$$\pi_{t-1} = \pi_{t-1}^c \tag{4}$$

where π_{t-1}^c indicates the cyclical inflation occurring in the period $t-1$. Replacing the right hand side of the (2) in the right hand side of the (3) and using the (4), follows then:

$$\pi_t - \vartheta \pi_{t-1}^c = -\alpha \left(-\beta g_{Y_t} \right) \tag{5}$$

Assuming to be interested to study the effects not of the GDP growth rate, but of the growth rate of a single component of the latter, that is of the net exports g_{NX_t}, the (5) becomes:

$$\pi_t - \vartheta\pi_{t-1}^c = -\alpha\left(-\beta g_{NX_t}\right)$$

namely:

$$\pi_t - \vartheta\pi_{t-1}^c = \alpha\beta\, g_{NX_t}$$

which, by setting $\alpha\beta = \gamma$, is rewrites as:

$$\pi_t - \vartheta\pi_{t-1}^c = \gamma\, g_{NX_t}$$

from which:

$$g_{NX_t} = \frac{1}{\gamma}\left(\pi_t - \vartheta\pi_{t-1}^c\right) \tag{6}$$

Recalling that net exports, are given by the difference between the exports (H_t) and the imports (Q_t):

$$NX_t = H_t - Q_t$$

it is possible to place:

$$H_t = h_0 + h_1 Y_t^* - h_2 r_t^* + h_3 C_t^* + h_4 G_t^* + h_5 P_t^* + h_6 \sin\left(g_{Y_t^*}\right)$$

$$Q_t = q_0 + q_1 Y_t - q_2 r_t + q_3 C_t + q_4 G_t + q_5 P_t + q_6 \sin\left(g_{Y_t}\right)$$

where Y_t is the GDP at time t, r_t is the real exchange rate (Real Exchange Rate) at time t, C_t represents the final consumption expenditure of private operators at time t, G_t is the final consumption expenditure of the public sector at time t, P_t is the price of the basket representative of the goods traded at time t, $\sin\left(g_{Y_t}\right)$ represents the sine of the GDP growth rate at time t. This latter term reveals, in both equations, the effect of the economic cycle on the exports and on the imports. The apex "*" in H_t indicates that it refers to the generic trading partner variable.

Calculating the growth rate g_{NX_t} as the sum of the partial first derivatives of the term $H_t - Q_t$ with respect to each variable, the following final relation is obtained:

$$g_{NX_t} = h_1 + h_2 + h_3 + h_4 + h_5 + h_6 \cos\left(g_{Y_t^*}\right)$$
$$- q_1 - q_2 - q_3 - q_4 - q_6 \cos\left(g_{Y_t}\right)$$

Indicating with τ the sum of all the constants h_i and q_i and replacing $\frac{1}{\gamma}\left(\pi_t - \vartheta\pi_{t-1}^c\right)$ instead of g_{NX_t}, the (6) becomes:

$$\left(\pi_t - \vartheta\pi_{t-1}^c\right) = \gamma\left\{\tau + h_6 \cos\left(g_{Y_t^*}\right) - q_6 \cos\left(g_{Y_t}\right)\right\} \tag{7}$$

This final relation indicates the existence of a linear relationship between the national inflation gap and the difference between the cosine of the GDP growth rate of the partner considered and the cosine of the national GDP growth rate. The difference to the second member of the (7) takes on a positive or a negative value depending on the value assumed by the elasticity of exports to the cosine of foreign real GDP growth rate (h_6) and depending on the value assumed by the elasticity of imports to the cosine of the real GDP growth rate of the national economy (q_6).

Assuming $h_6 > 0$ and $q_6 < 0$, an increase in the effective national inflation extends the difference to the second member of the (7), namely the difference in growth between the national economy and the economy of the trading partner, due to the effects of the economic cycle on the foreign economy and on the national economy. On the other hand, an increase in national expected inflation leads to a reduction in the growth differential between the national economy and the foreign economy, due to the effects of the economic cycle on the foreign economy and the national economy.

Assuming $h_6 < 0$ and $q_6 > 0$, an increase in the national effective inflation reduces the real GDP growth rate of the foreign economy, due to the effects of the economic cycle, and increases the real GDP growth rate of the national economy due to the effects of the economic cycle. An increase in national expected inflation has, instead, the exact opposite effect.

Assuming $h_6 > 0$ and $q_6 > 0$, an increase in national effective inflation increases the real GDP growth rate, due to the effects of the economic cycle, in both economies considered while an increase in the national expected inflation involves a reduction in the real GDP growth rate, due to the effects of the economic cycle in both economies.

Assuming $h_6 < 0$ and $q_6 < 0$, an increase in the national effective inflation has a negative effect both on the real GDP growth rate of the national economy, due to the effects of the economic cycle, and on the real GDP growth rate of the foreign economy, due again to the effects of the economic cycle.

Assuming $h_6 = 0$ and $q_6 \neq 0$, the effective national inflation and the national expected inflation have effect only on the real GDP growth rate, due to the effects of the business cycle, of the trading partner. The sign of this effect depends on the sign of q_6.

Assuming $h_6 \neq 0$ and $q_6 = 0$, the effective national inflation and national expected inflation have effect only on the real GDP growth rate, due to the effects of the business cycle, of the national economy. The sign of this effect depends on the sign of h_6.

Assuming $h_6 = q_6 = 0$, the relationship is no longer valid.

In the following paragraph, are estimated the export and the import functions of Germany with respect to the complex of the countries belonging to the euro area; more precisely, the authors proceed estimating the export and the import function of Germany, with respect to the countries of the euro area, through the ordinary least squares method (OLS), using standard errors robust to heteroskedasticity and autocorrelation (HAC) and placing both the dependent variable and the regressors in logarithm in order to

estimate the elasticities of the dependent variables with respect to each respective regressor.

For the exports the theoretical model to be estimated is the following:

$$\log(H_t) = \hat{\beta}_0 + \hat{\beta}_1 \log(Y_t^*) + \hat{\beta}_2 \log(r_t^*) + \hat{\beta}_3 \log(C_t^*) + \hat{\beta}_4 \log(G_t^*) + \hat{\beta}_5 \log(P_t^*) + \hat{\beta}_6 \left(\sin\left(g_{Y_t^*}\right)\right) + \hat{u}_t$$

where all the variables are considered in real terms while $\hat{u}_t$ is the residuals's term.

For the imports the theoretical model to be estimated is following:

$$\log(Q_t) = \hat{\delta}_0 + \hat{\delta}_1 \log(Y_t) + \hat{\delta}_2 \log(r_t) + \hat{\delta}_3 \log(C_t) + \hat{\delta}_4 \log(G_t) + \hat{\delta}_5 \log(P_t) + \hat{\delta}_6 \left(\sin\left(g_{Y_t}\right)\right) + \hat{z}_t$$

where all the variables are considered in real terms while $\hat{z}_t$ is the residuals's term.

The terms $\sin\left(g_{Y_t^*}\right)_e$ $\sin\left(g_{Y_t}\right)$ indicate, respectively, the effect of the business cycle on the real GDP of the euro area and on the German real GDP and correspond to the smoothed components of the historical series, relative to the real GDP growth rate of the euro area and of the real German GDP and are estimated using the special Hodrick-Prescott filter and setting $\lambda = 1600$.

Since it is complex to estimate the growth rate of elasticity compared to single export and import variables in each lag t, the authors proceed estimating a single value for each variable (value representative of the entire sample period) and verifying its stability through the appropriate CUSUM test.

The authors also verify the empirical validity of the assumption $\pi_{t-1} = \vartheta \pi_{t-1}^c$ and they estimate the value ϑ through an ARMAX model (Autogressive Moving Average with exogenous inputs). The dependent variable is the first difference of the effective inflation rate of Germany, *d_InfCPIGer* and the only regressor is the first difference of the cyclical component of the inflation rate, *d_CyclicalInfCPIGer*. This last variable was obtained by applying a Hodrick-Prescott filter to the historical series of effective inflation at the CPI on a quarterly basis and not seasonally adjusted, by setting $\lambda = 1600$.

After estimating the elasticity of German exports and imports compared to each variable and after verifying the validity of the assumptions related to the expected inflation rate, the authors verify the existence and the significance of the final relation through a Vector Autoregression model of order p, VAR(p), in order to obtain estimated values of γ for the different periods t. The VAR(p) can be written as follows:

$$SmoothedInfCPIGer = c_1 + a_{11}\, SmoothedInfCPIGer_{t-1} + \ldots + a_{15}\, SmoothedInfCPIGer_{t-5} + b_{11}\, CyclicalGrowthEur_{t-1} + \ldots + b_{15}\, CyclicalGrowthEur_{t-5} + \varepsilon_{1t}$$

$$CyclicalGrowthEur = c_2 + a_{21}\, CyclicalGrowthEur_{t-1} + \ldots + a_{25}\, CyclicalGrowthEur_{t-5} + b_{21}\, SmoothedInfCPIGer_{t-1} + \ldots + b_{25}\, SmoothedInfCPIGer_{t-5} + \varepsilon_{2t}$$

where every estimated coefficient (b_{11},…, b_{15}) and (b_{21},…, b_{25}) corresponds to an estimated value of γ paramter at the time $t-k$. The only two endogenous variables considered are *Inflation DifferenceGer* and *CyclicalInfCPIGer*. The first variable corresponds to the difference to the first member of the (7).

4. ESTIMATION OF THE MODEL AND ANALYSIS OF THE RESULTS

All the variables indicated in the previous paragraph are on a quarterly basis and were considered in the sample period Q1 1999 - Q1 2018. The variables used to estimate the export and the import function of Germany are integrated of order 3 according to the Augmented Dickey-Fuller test (ADF) performed with BIC criteria and testing down from maximum lag order 11. The same variables are not cointegrated based on the Engle-Granger cointegration test performed without constant and testing down from maximum lag order 10.

The estimated model for exports is as follows:

Model 1: OLS, using observations 1999:4-2018:1 (T = 74) Dependent variable: d_d_d_l_RealExpGer HAC standard errors, bandwidth 3 (Bartlett kernel)				
	Coefficient	Std. Error	t-ratio	p-value
const	0.00408939	0.00492556	0.8302	0.4094
d_d_d_l_RealGDPEur	5.77017	0.931671	6.193	<0.0001***
d_d_d_l_RealPriFinConsExpEur	−6.32622	0.938415	−6.741	<0.0001***
d_d_d_l_RealGvtFinConsExpEur	0.345691	0.100339	3.445	0.0010***
d_d_d_l_ExportPriceGer	−1.21762	1.06717	−1.141	0.2579
d_d_d_l_RERGer	−9.19319	1.47073	−6.251	<0.0001***
d_d_d_l_hpt_Growth Eur	−12.7621	4.85009	−2.631	0.0105**

Mean dependent var	0.001725	S.D. dependent var	0.138611
Sum squared resid	0.330098	S.E. of regression	0.070191
R-squared	0.764644	Adjusted R-squared	0.743568
F(6, 67)	38.48559	P-value(F)	7.32e-20
Log-likelihood	95.25843	Akaike criterion	−176.5169
Schwarz criterion	−160.3884	Hannan-Quinn	−170.0830
rho	−0.271975	Durbin-Watson	2.467460

RESET test for specification -
Null hypothesis: specification is adequate Test statistic:
F(2, 65) = 0.173413
with P-value = P(F(2, 65) > 0.173413) = 0.841178
White's test for heteroskedasticity -
Null hypothesis: heteroskedasticity not present Test statistic:
LM = 28.1135

with P-value = P(Chi-square(27) > 28.1135) = 0.405138
Test for normality of residual -
Null hypothesis: error is normally distributed Test statistic: Chi-square(2) = 2.0747
with P-value = 0.354393
Test for ARCH of order 4 -
Null hypothesis: no ARCH effect is present Test statistic: LM = 2.74489
with P-value = P(Chi-square(4) > 2.74489) = 0.601383
CUSUM test for parameter stability - Null hypothesis: no change in parameters
Test statistic: Harvey-Collier t(66) = −0.436896 with P-value = P(t(66) > −0.436896) = 0.663613
The Jarque-Bera test does not reject the null hypothesis of normal distribution of the residuals.
The F test for zero slopes rejects the null hypothesis for which all the estimated coefficients are statistically not significant.
The White test does not reject the null hypothesis of the absence of heteroskedasticity in the residuals.
The ARCH test does not reject the null hypothesis of the absence of serial autocorrelation in the squares of the residuals.
Ramsey's RESET test does not reject the null hypothesis of linear specification of the regression model.
The CUSUM test does not reject the null hypothesis of the absence of structural breaks, as shown in Figure 1:

All the estimated coefficients are significantly different from 0, with the exception of the constant $\hat{\beta}_0$ and that relating to the export price index $\hat{\beta}_5$. Note that the coefficient $\hat{\beta}_6$ has a negative and very high value.

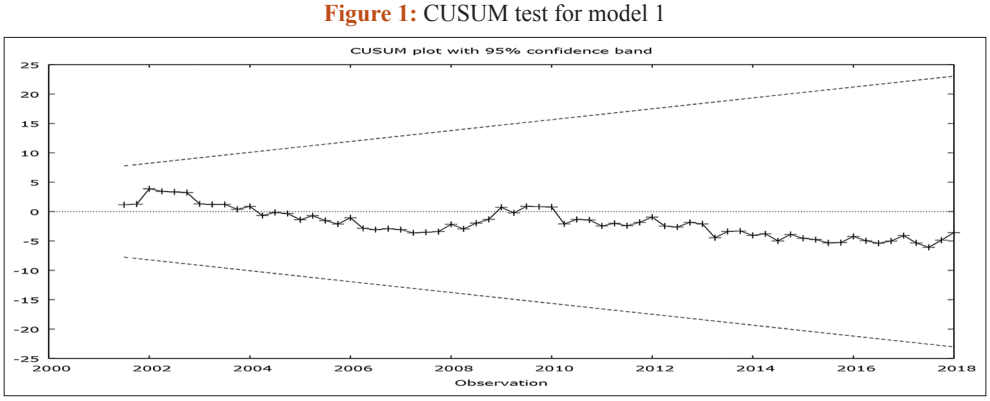
The estimated model for imports is the following:

Model 2: OLS, using observations 1999:4-2018:1 (T = 74) Dependent variable: d_d_d_l_RealImpGer HAC standard errors, bandwidth 3 (Bartlett kernel)				
	Coefficient	SE	t-ratio	p-value
const	0.00258563	0.00457315	0.5654	0.5737
d_d_d_l_RealGDPGer	0.841306	0.516434	1.629	0.1080
d_d_d_l_RealPriFinConsExpGer	−0.331251	0.331517	−0.9992	0.3213
d_d_d_l_RealGvtFinalConsExpGe	0.377615	0.115945	3.257	0.0018***
d_d_d_l_ImportPriceGer	1.45503	0.544972	2.670	0.0095***
d_d_d_l_REREur	6.15321	2.27826	2.701	0.0088***
d_d_d_l_hpt_GrowthG er	−1.60400	1.76099	−0.9109	0.3656

Mean dependent var	0.001332	S.D. dependent var	0.152407
Sum squared resid	0.749514	S.E. of regression	0.105768
R-squared	0.557975	Adjusted R-squared	0.518390
F(6, 67)	7.086307	P-value(F)	7.17e-06
Log-likelihood	64.91719	Akaike criterion	−115.8344
Schwarz criterion	−99.70592	Hannan-Quinn	−109.4005
rho	−0.636643	Durbin-Watson	3.192500

RESET test for specification -
Null hypothesis: specification is adequate Test statistic: F(2, 65) = 0.705768
with P-value = P(F(2, 65) > 0.705768) = 0.497473
White's test for heteroskedasticity -
Null hypothesis: heteroskedasticity not present Test statistic: LM = 28.423
with P-value = P(Chi-square(27) > 28.423) = 0.389417
Test for normality of residual -
Null hypothesis: error is normally distributed Test statistic: Chi-square(2) = 0.0219212 with P-value = 0.989099
Test for ARCH of order 4 -
Null hypothesis: no ARCH effect is present Test statistic: LM = 20.3255
with P-value = P(Chi-square(4) > 20.3255) = 0.000430662
CUSUM test for parameter stability - Null hypothesis: no change in parameters
Test statistic: Harvey-Collier t(66) = −0.266864 with p-value = P(t(66) > −0.266864) = 0.790406
The Jarque-Bera test does not reject the null hypothesis of normal distribution of the residuals.
The F test for zero slopes rejects the null hypothesis for which all the estimated coefficients are statistically not significant.
The White test does not reject the null hypothesis of the absence of heteroskedasticity in the residuals.

Figure 1: CUSUM test for model 1



The ARCH test rejects the null hypothesis of the absence of serial autocorrelation in the squares of the residuals. However, this is not a problem thanks to the use of HAC errors.
Ramsey's RESET test does not reject the null hypothesis of linear specification of the regression model. The CUSUM test does not reject the null hypothesis of the absence of structural breaks, as shown in Figure 2:

In this case only the coefficients $\hat{\delta}_2$, $\hat{\delta}_4$ and $\hat{\delta}_5$ are significantly different from 0, while all others, including $\hat{\delta}_6$, are not significantly different from 0.

The values of $\hat{\beta}_6$ and $\hat{\delta}_6$ indicate that the real German exports feel strongly and negatively the effects of the economic cycle relative to the countries of the euro area and that the real German imports are rigid with respect to the effects of the economic cycle on the national GDP. This could be explained by looking at the timeplot of the variables *hpt_GrowthEur* and *hpt_GrowthGer* in Figure 3:

It is noted, in fact, that the first has a periodic wave trend that has an higher amplitude in the first part of the sample and a less amplitude in the second part. In other words, the periodic function seems to follow a decreasing trend, indicating that in the time interval considered the economic cycle has an overall negative effect on the GDP of the euro area. The second seems, instead, a periodic wave with an increasing trend but with a generally low amplitude. This second result could explain the reason why German real GDP is not particularly affected by the economic cycle.

The null hypothesis in the CUSUM tests relating to the two OLS models is not rejected with a p-value much greater than the significance level $\alpha = 0.05$. This indicates that the estimated parameters are stable over time and do not present significant changes. Therefore, the estimated elasticities can be approximated to constants respect to the time.

The three variables used in the estimation of the ARMAX model are integrated of order 4 according to the ADF test carried out with the same methods used for the two previous OLS models and are not cointegrated according to the results of the Engle-Granger test again performed with the methods indicated above.

Before proceeding with the estimation, the correlogram of the residuals (Figure 4) of the variable *d_InfCPIGer*, on 16 lag, was observed to establish how many autoregressive (AR) and moving average (MA) components to set:

Here ACF indicates the global autocorrelations and PACF indicates the partial autocorrelations. The latter are estimated using the Durbin-Levinson algorithm (Levinson, 1946), (Durbin, 1960). On the base of this result an ARMAX(0, 1, 0) was estimated by setting *d_InfCPIGer* as a dependent variable, *d_CyclicalInfGer_1* namely the cyclical inflation rate of Germany lagged of 1 period, as an exogenous input and inserting a constant. It represents the first difference of the *CylicalInfGer* variable at time $t-1$ and was obtained by modeling the original series as an AR(1) process. The estimated ARMAX(0, 1, 0) is the following:

Figure 2: CUSUM test for model 2

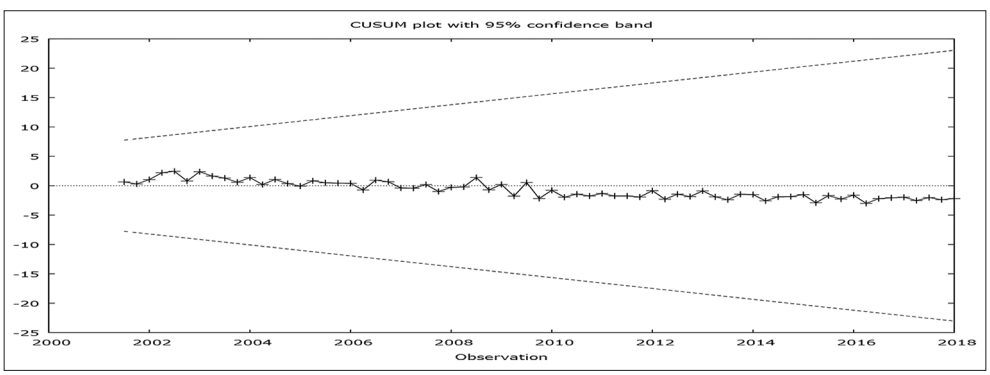


Figure 3: Smoothed component of the real GDP growth rate for euro area and Germany estimated through the HP filter

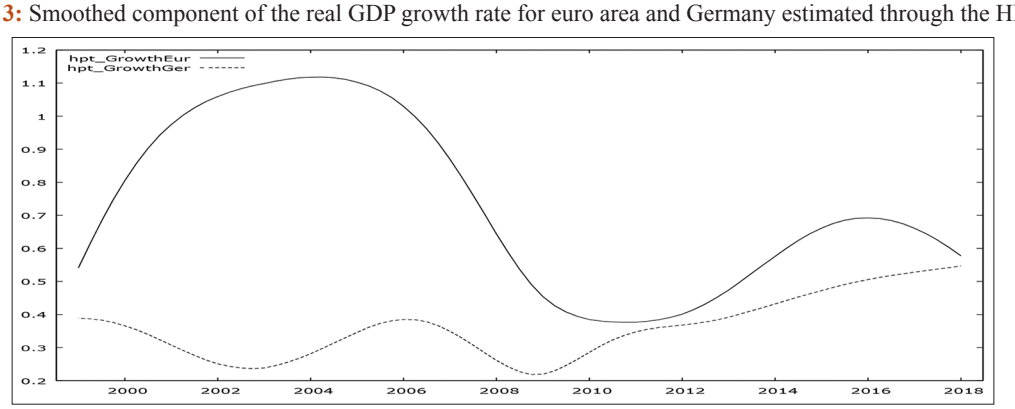
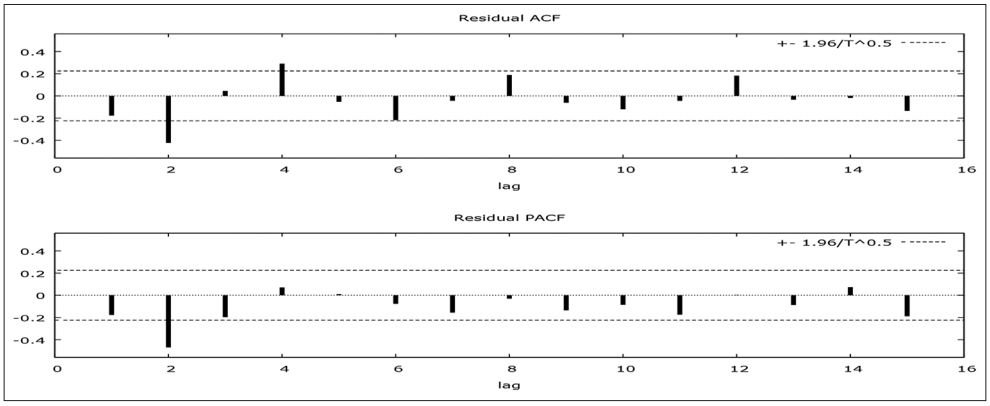


Figure 4: Residual correlogram of the variable *d_InfCPIGer*



Model 3: ARMAX, using observations 1999:2-2018:1 (T = 76)				
Dependent variable: d_InfCPIGer				
	Coefficient	SE	z	p-value
const	0.00602759	0.0459438	0.1312	0.8956
d_CyclicalInfGert_1	−0.418562	0.105614	−3.963	<0.0001***

Mean dependent var	0.002268	S.D. dependent var	0.437948
Mean of innovations	0.000000	S.D. of innovations	0.400444
Log-likelihood	−37.27209	Akaike criterion	80.54418
Schwarz criterion	87.53638	Hannan-Quinn	83.33860

Test for normality of residual -
Null hypothesis: error is normally distributed
Test statistic: Chi-square(2) = 3.85951
with p-value = 0.145184
LM test for autocorrelation up to order 4 -
Null hypothesis: no autocorrelation
Test statistic: Chi-square(4) = 23.9425
Test for ARCH of order 4 -
Null hypothesis: no ARCH effect is present
Test statistic: LM = 1.80277
with p-value = P(Chi-square(4) > 1.80277) = 0.771975

As can be seen, the estimated parameter associated with the independent variable is negative and significantly different from 0 with size $= 0,01$.

The Jarque-Bera test does not reject the null hypothesis of normal distribution of the residuals.

The LM test does not reject the null hypothesis of absence of autocorrelation in the residuals up to lag 4.

The ARCH test up to lag 4 does not reject the null hypothesis of the absence of conditional heteroskedasticity in the squared residuals.

The diagnostic tests therefore indicate that the coefficient −0.418562 represents a consistent and undistorted estimate of the parameter ϑ. Based on all the previous results, the final relation of the model in the case of the trade between Germany and the euro area countries takes the following form:

$$\pi_t + 0,418562\pi_{t-1}^c = \gamma\left\{0,0246031 - 12,7621 hpt_GrowthEur_t\right\}$$

where 0,0246031 corresponds to the term $\tau = (\hat{\beta}_1 + \hat{\beta}_2 + \hat{\beta}_3 + \hat{\beta}_5 + \hat{\beta}_6) - (\hat{\delta}_3 + \hat{\delta}_4 + \hat{\delta}_5)$ and *hpt_GrowthEur* presents, as seen in Figure 3, a periodic trend. More precisely, an increase in the effective inflation rate in Germany leads to a slowdown in the cyclical growth rate of the real GDP in the countries of the euro area. An increase in the German expected inflation rate has the opposite effect. On the other hand, an increase in the cyclical growth rate of the real GDP in the countries of the euro area entails a reduction in the inflation gap. By this final relation are defined the variables *InflationDifferenceGer* and *CyclicalGrowthEur*, which respectively correspond to the left hand side and the right hand side of the relation itself.

Regarding the estimation of the VAR(p), the ADF test and Engle-Granger cointegration test, performed with the same methods used in the previous models, indicate that the two last variables are integrated of order 3 and are not cointegrated. The estimation of the p to assign to the VAR(p) is selected by minimizing the likelihood function given the two variables including a constant and setting 10 lag as maximum, on the basis of the criteria of Akaike (AIC), Schwarz (BIC) and Hannan-Quinn (HQC). According to the BIC and HQC criteria the optimal value is 6, instead according to AIC it is 8:

VAR system, maximum lag order 10

The asterisks below indicate the best (that is, minimized) values of the respective information criteria, AIC = Akaike criterion, BIC = Schwarz Bayesian criterion and HQC = Hannan-Quinn criterion.

	lagsloglik	p(LR)	AIC	BIC	HQC
1	89.56892	−2.611529	−2.409133	−2.531795	
2	115.05485	0.00000	−3.282964	−2.945639	−3.150075
3	132.34681	0.00000	−3.698338	−3.226082	−3.512292
4	159.49455	0.00000	−4.421705	−3.814519	−4.182503
5	206.19189	0.00000	−5.755997	−5.013881	−5.463640
6	216.09609	0.00054	−5.940503	−5.063457*	−5.594990*
7	220.61128	0.06034	−5.956603	−4.944626	−5.557934
8	225.02278	0.06568	−5.969462*	−4.822555	−5.517637
9	228.50431	0.13785	−5.953260	−4.671423	−5.448280
10	230.22040	0.48827	−5.881887	−4.465120	−5.323751

The authors prefer to choose $p = 6$. The estimated VAR(6) is the following:

VAR system, lag order 6
OLS estimates, observations 2001:2-2018:1 (T = 68)
Log-likelihood = 227.33075
Determinant of covariance matrix = 4.2783097e-006
AIC = -5.9215
BIC = -5.0729
HQC = -5.5852
Portmanteau test: LB(17) = 53.9464, df = 44 [0.1447]

Equation 1: d_d_d_CyclicalGrowthEur
HAC standard errors, bandwidth 3 (Bartlett kernel)

const	Coefficient	SE	t-ratio	p-value
	−0.000270582	0.000683089	−0.3961	0.6936
d_d_d_Cyclical GrowthEur_1	0.902462	0.127133	7.099	<0.0001***
d_d_d_Cyclical GrowthEur_2	−0.175604	0.113811	−1.543	0.1286
d_d_d_Cyclical GrowthEur_3	−0.165081	0.0397420	−4.154	0.0001***
d_d_d_Cyclical GrowthEur_4	0.839915	0.0423066	19.85	<0.0001***
d_d_d_Cyclical GrowthEur_5	−0.920275	0.125182	−7.352	<0.0001***
d_d_d_Cyclical GrowthEur_6	0.150855	0.104110	1.449	0.1530
d_d_d_Inflation DifferenceGer_1	0.00318391	0.00145896	2.182	0.0334**
d_d_d_Inflation DifferenceGer_2	0.00467455	0.00256297	1.824	0.0736*
d_d_d_Inflation DifferenceGer_3	0.00675593	0.00321957	2.098	0.0405**
d_d_d_Inflation DifferenceGer_4	0.00370348	0.00356183	1.040	0.3030
d_d_d_Inflation DifferenceGer_5	0.00207904	0.00273564	0.7600	0.4505
d_d_d_Inflation DifferenceGer_6	0.00114524	0.00166657	0.6872	0.4949

Mean dependent var	−0.000889	S.D. dependent var	0.018535
Sum squared resid	0.002147	S.E. of regression	0.006247
R-squared	0.906742	Adjusted R-squared	0.886394
F(12, 55)	86.44743	P-value(F)	3.44e-31
rho	0.009549	Durbin-Watson	1.926316

F-tests of zero restrictions:
All lags of d_d_d_CyclicalGrowthEurF(6, 55) = 109.39 [0.0000]
All lags of d_d_d_InflationDifferenceGerF(6, 55) = 3.7321 [0.0035]
All vars, lag 6 F(2, 55) = 2.9502 [0.0607]

Equation 2 d_d_d_InflationDifferenceGer
HAC standard errors, bandwidth 3 (Bartlett kernel)

const	Coefficient	SE	t-ratio	p-value
	−0.0157449	0.0362567	−0.4343	0.6658
d_d_d_Cyclical GrowthEur_1	17.6355	6.37583	2.766	0.0077***
d_d_d_Cyclical GrowthEur_2	−7.26345	6.29837	−1.153	0.2538
d_d_d_Cyclical GrowthEur_3	−8.07661	4.26194	−1.895	0.0633*
d_d_d_Cyclical GrowthEur_4	−4.18142	2.52778	−1.654	0.1038
d_d_d_Cyclical GrowthEur_5	−13.7786	6.11886	−2.252	0.0283**
d_d_d_Cyclical GrowthEur_6	17.0891	5.68078	3.008	0.0040***
d_d_d_Inflation DifferenceGer_1	−1.77026	0.104465	−16.95	<0.0001***
d_d_d_Inflation DifferenceGer_2	−2.47561	0.186909	−13.24	<0.0001***
d_d_d_Inflation DifferenceGer_3	−2.52463	0.260951	−9.675	<0.0001***
d_d_d_Inflation DifferenceGer_4	−2.01616	0.226327	−8.908	<0.0001***
d_d_d_Inflation DifferenceGer_5	−1.14134	0.142186	−8.027	<0.0001***
d_d_d_Inflation DifferenceGer_6	−0.418785	0.0742450	−5.641	<0.0001***

Mean dependent var	−0.009468	S.D. dependent var	0.999014
Sum squared resid	10.63420	S.E. of regression	0.439715
R-squared	0.840967	Adjusted R-squared	0.806269
F(12, 55)	66.69191	P-value(F)	2.68e-28
rho	−0.137301	Durbin-Watson	2.259177

F-tests of zero restrictions:
All lags of d_d_d_CyclicalGrowthEurF(6, 55) = 4.5822 [0.0008]
All lags of d_d_d_InflationDifferenceGerF(6, 55) = 90.674 [0.0000]
All vars, lag 6 F(2, 55) = 16.93 [0.0000]

For the system as a whole
Null hypothesis: the longest lag is 5
Alternative hypothesis: the longest lag is 6
Likelihood ratio test: Chi-square(4) = 18.0594 [0.0012]

In this case all the autocorrelations in the series of the residuals are significantly different from 0 both for the Portmanteau test indicated above, and for the Rao F test up to the order 4:

Test for autocorrelation of order up to 4

	Rao F	Approx dist.	p-value
lag 1	2.202	F(4, 104)	0.0739
lag 2	1.801	F(8, 100)	0.0855
lag 3	1.673	F(12, 96)	0.0848
lag 4	1.381	F(16, 92)	0.1686

Furthermore, the Doornik-Hansen test does not reject the null hypothesis of normal distribution of residuals:

Residual correlation matrix, C (2 × 2)

1.0000	0.36519
0.36519	1.0000

Eigenvalues of C
0.63481
1.36519
Doornik-Hansen test
Chi-square(4) = 4.0325 [0.4016]

The ARCH test up to lag 4 does not reject the null hypothesis of absence of autocorrelation in the series of squares of residuals.

Infact:

Test for ARCH of order up to 4

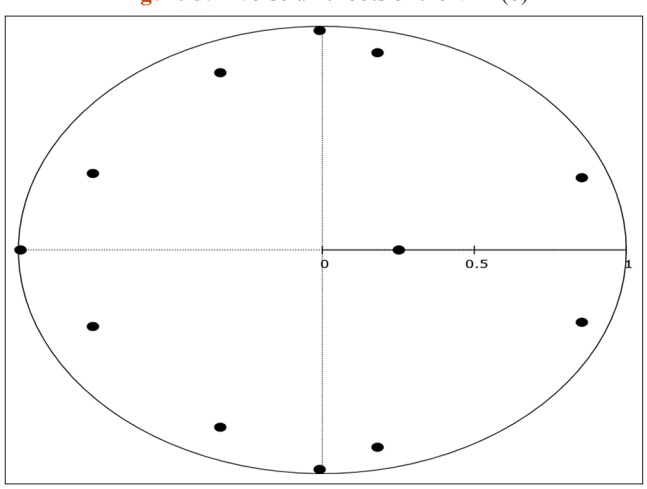
	LM	df	p-value
lag 1	8.796	9	0.4564
lag 2	12.080	18	0.8431
lag 3	32.451	27	0.2158
lag 4	37.915	36	0.3820

In both equations, the F test for zero slopes rejects the null hypothesis that all the estimated coefficients are significantly different from 0. The F test of zero restrictions, in both equations, rejects the null hypothesis that all the estimated coefficients associated with the variable *d_d_d_InflationDifferenceGer* and all the estimated coefficients associated with the variable *d_d_d_CyclicalGrowthEur* are null. The appropriate graphic analysis in Figure 5 finally indicates that all the inverse roots of the autoregressive component are included in the unit circle:

The three outermost roots on the unit circle have the following coordinates: (−0,01; 0,98), (0,01; 0,98), (0,00; −0,99). All the roots are included in the unit circle and thus the stationarity of the VAR(6) is verified.

In the first equation of the VAR(6) the estimated coefficients *d_d_d_InflationDifferenceGer_1, d_d_d_InflationDifferenceGer_2, d_d_d_InflationDifferenceGer_3* are significantly different from 0 and has positive sign. In general, the standard deviation associated with the dependent variable *d_d_d_CyclicalGrowthEur* is very low, demonstrating that the estimated coefficients have a large size. With regard to the size of the coefficients, note also that the variation of time alternates increases and reductions. This is due to the fact that the effect of a marginal variation in the German inflation gap on the cyclical GDP growth rate of the euro area depends on the cyclical inflation, namely on the phases of the economic cycle. In order to verify the latter assumption, an Impulse response function (IRF) was estimated on a forecast horizon of 38 lags (maximum forecast horizon), placing the variable *d_d_d_InflationDifferenceGer* next the variable *d_d_d_CyclicalGDPGrowthEur* and simulating in this way the effect on the variable *d_d_d_InflationDifferenceGer* of a shock coming from the variable *d_d_d_CyclicalGDPGrowthEur*.

Figure 5: Inverse unit roots of the VAR(6)



The result is shown in Figure 6:

An increase in the inflation gap has an initially positive effect in the first 3 lags of the forecast horizon. Subsequently, throughout the sample, positive and negative changes alternate and repeat themselves over time.

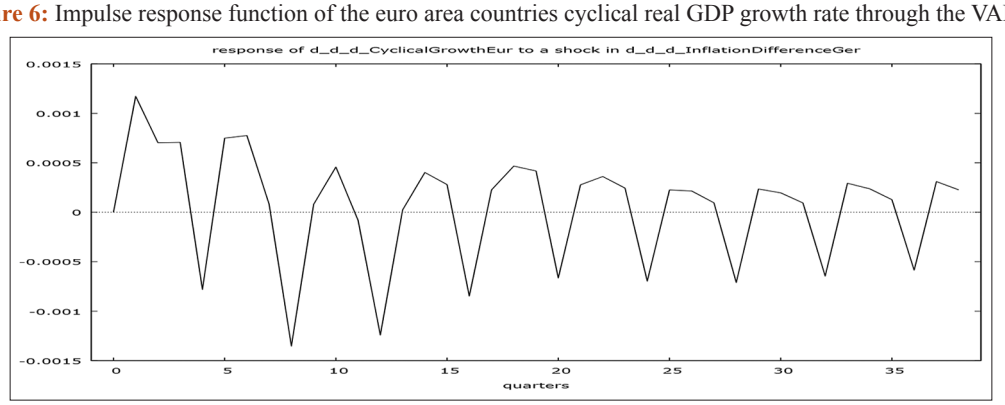
In the second equation of the VAR(6) the coefficients *d_d_d_CyclicalGrowthEur*_1, *d_d_d_ CyclicalGrowthEur_3, d_d_d_CyclicalGrowthEur_5* and *d_d_d_ CyclicalGrowthEur_6* are significantly different from 0 and alternate positive and negative sign. In this case the standard deviation of the dependent variable *d_d_d_InflationDifferenceGer* is close to 1.

After the authors perform an IRF of the variable *d_d_d_InflationDifferenceGer* against shocks coming from the variable *d_d_d_CyclicalGrowthEur,* respecting the same order as before between the two variables. The results obtained is illustrated in Figure 7:

As can be seen in the first 7 quarters, the response of the inflation gap to the shocks is negative, instead the rest of the sample alternates positive and negative changes. Overall, the impulse response function seems to follow a periodic trend.

It should also be noted that the adjusted R square is very close to 1 and this allows the model to be used to make a statistic forecast of the *d_d_d_CyclicalGrowthEur* variable, given the variable

Figure 6: Impulse response function of the euro area countries cyclical real GDP growth rate through the VAR(6)



d_d_d_InflationDifferenceGer, in order to evaluate its forecasting capacity. The forecast range chosen is Q2 2000 - Q1 2018.

The result is shown in Figure 8:

Forecast evaluation statistics	
Mean Error	-2.0727e-020
Root Mean Squared Error	0.0056185
Mean Absolute Error	0.0043978
Mean Percentage Error	5.3189
Mean Absolute Percentage Error	70.368
Theil's U	0.19622
Bias proportion, UM	0
Regression proportion, UR	0
Disturbance proportion, UD	1

The mean forecast error is close to 0. From its decomposition it results that the disturbance proportion (UD), the regression proportion (UR) and the bias proportion (UM) are at their optimal value, i.e. 1, 0 and 0 respectively. The Theil coefficient is equal to 0.19. These values indicate a good capacity of the model to predict the cyclical GDP growth rate of the euro area's countries.

Finally the authors make a static prediction of the variable *d_d_d_InflationDifferenceGer,* given the variable *d_d_d_CyclicalGrowthEur*. The result is shown in Figure 9:

Figure 7: Impulse response function of the German inflation gap through the VAR(6)

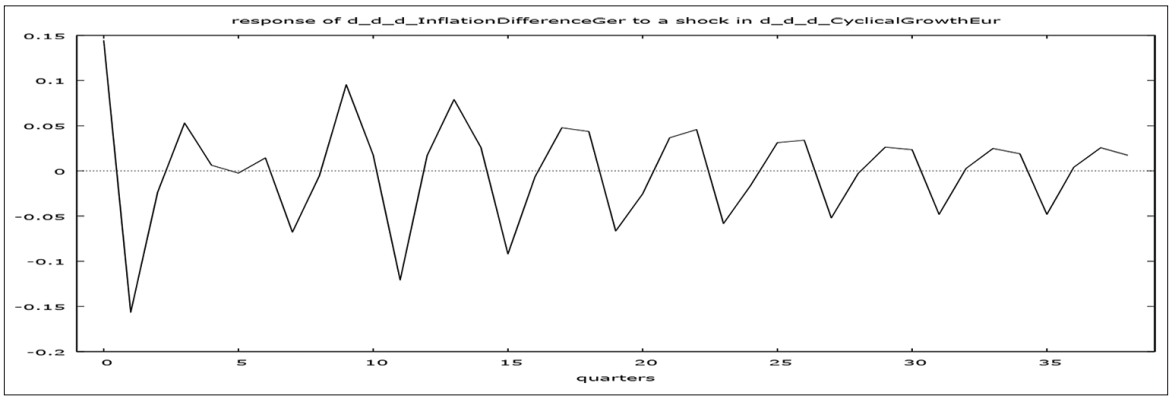


Figure 8: Static forecast of the cyclical real GDP growth rate of euro area through the German inflation gap

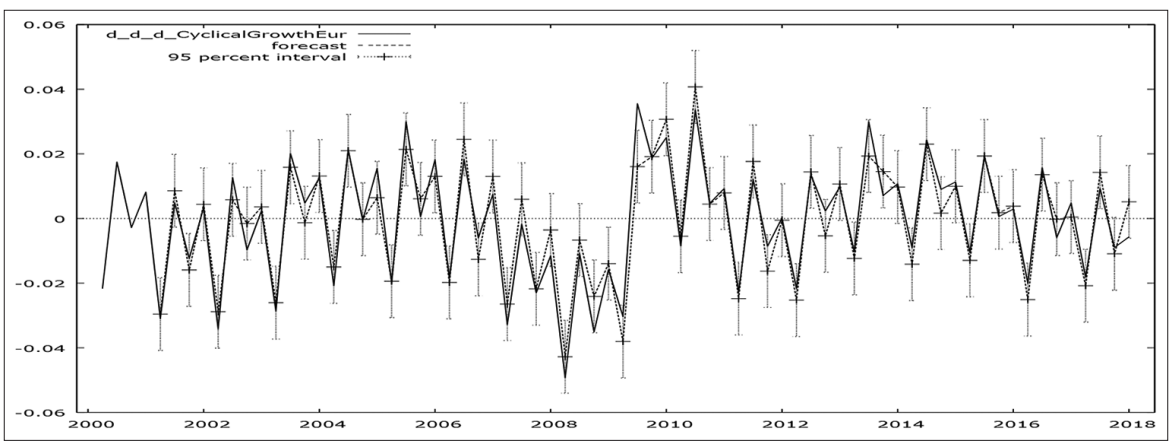
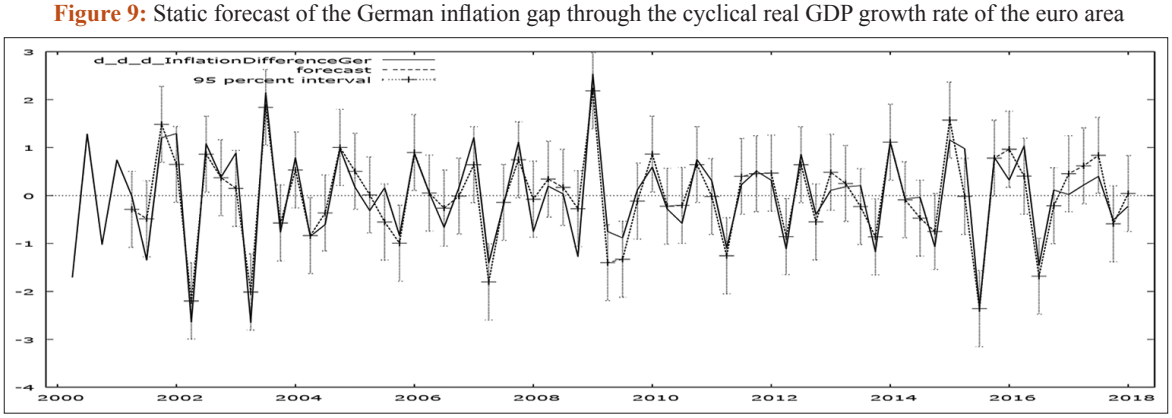


Figure 9: Static forecast of the German inflation gap through the cyclical real GDP growth rate of the euro area



Forecast evaluation statistics	
Mean Error	2.6276e-017
Root Mean Squared Error	0.39546
Mean Absolute Error	0.31545
Mean Percentage Error	-69.965
Mean Absolute Percentage Error	148.91
Theil's U	0.65869
Bias proportion, UM	0
Regression proportion, UR	0
Disturbance proportion, UD	1

Also in this case the values resulting from the decomposition of the absolute error are at their optimal value, however the Theil coefficient is very high (0.65). For this reason the VAR(6) cannot be considered a good predictive model for the German inflation gap.

5. CONCLUSIONS

The empirical results presented in the paragraph 3 indicate the validity of the conclusion according to which the real German exports depend only on the phases of the economic cycle in the countries of the euro area. More precisely, the authors pointed out that, in the case of Germany, the exports towards the countries of the euro area depend negatively on the effects of the economic cycle, while the imports from the countries belonging to the European single currency are rigid compared to the latter. For this reason, in the case of Germany and of the countries of the European single currency, appears, with a negative sign, only the cosine of the growth rate of the real GDP of the euro area due to the effects of the economic cycle.

It is also empirically founded, with high evidence (as indicated by the low p-value of the ARMAX model), the assumption according to which the cyclical component of the inflation rate, recorded in the period $t-1$ can be used to predict the effective inflation, (and therefore the economic operators in Germany could formulate their expectations through the same, i.e. taking into account more closely the short-term changes that typically involve consumer goods such as foodstuffs and electricity).

Since the estimated coefficient is negative, it is assumed that German economic operators associate, with an increase in cyclical inflation at period $t-1$, a reduction of the effective inflation in the subsequent period.

The final linear relation, to which the authors arrive, is empirically verified based on the results of the VAR(6). The dimension and the sign of the effect, on the inflation gap, of the real GDP growth rate, due to the effects of the economic cycle, depend on the phases of the economic cycle itself and on how the different elasticities of the exports and of the imports vary respect to the time t. In the case examined by the authors, however, observing the sign of the estimated coefficients, the German inflation gap reacts to changes in the real growth rate of real GDP in the euro area behaving similarly to a periodic wave, exhibiting, that is, a periodic trend. On the other hand, an increase of the real GDP growth rate, due to the effects of the economic cycle, in the euro area always has a positive effect on the inflation gap.

The IRF reported in Figure 6 shows that a shock of the real GDP growth rate, in the euro area, due to the effects of the cycle at time t, involves the alternation of negative and positive changes in the German inflation gap, depending on whether the euro area real GDP is in an expansionary or depressive phase of the economic cycle. These changes seem to fade over time as a consequence of the fact that gradually the real euro area GDP tends to its natural level.

The IRF exposed in Figure 7 shows that a shock of the inflation gap at time t leads positive and negative changes of the growth rate of the euro area real GDP, due to the effects of the cycle. The function seems to follow a periodic and regular pattern during the sample.

The results of the static forecasts exposed in Figure 8 and in Figure 9 indicate that the model is useful to predict, with good precision, the real GDP growth rate, due to the effects of the euro area economic cycle, given the German inflation gap, but it is not able to forecast, with good approximation, the German inflation gap through the real GDP growth rate of the euro area, due to the effects of the economic cycle.

The authors conclude hoping that the elaborated and the estimated model, even in their simplicity, can represent a useful analysis tool for the scholars of economic sciences and of economic policy, for political decision-makers and that it may arouse interest also for scholars of the Theory of International Trade. In fact, according to the authors' results, the phases of the economic cycle determine changes in the inflation gap and, consequently, changes in the competitiveness of the national economy towards the foreign economy.

REFERENCES

Atkenson, A., Ohanian, L.E. (2001), Are Phillips Curves useful for forecasting inflation? Federal Reserve Bank of Minneapolis Quarterly Review, 25(1), 2-11.

Durbin, J. (1960), The fitting of time-series models. Review of International Statistical Institute, 28(3), 233-244.

Fischer, S. (1977), Long-term contracts, rational expectations and the optimal money supply rule. Journal of Political Economy, 85, 191-205.

Friedman, M. (1968), The role of monetary policy. American Economic Review, 58(1), 1-17.

Levinson, N. (1946), The Wiener (Root Mean Square) error criterion in filter design and prediction. Studies in Applied Mathematics, 25, 261-278.

Lucas, R.E. (1976), Econometric policy evaluation: A critique. Carnegie-Rochester Conference Series on Public Policy, 1, 19-46.

Lucas, R.E., Sargent, T. (1979), After Keynesian macroeconometrics. Quarterly Review, 3(2), 1-16.

Okun, A.M. (1962), Potential GNP: Its Measurement and Significance. Cowles Foundation Paper, No. 190. p1-7.

Phelps, E.S. (1967), Expectations of inflation and optimal unemployment over time. Economica, 34(135), 254-281.

Phelps, E.S. (1968), Money-Wage dynamics and labor-market equilibrium. Journal of Politcal Economy, 76(4), 678.

Phillips, W. (1958), The relation between unemployment and the rate of change of money wage rates in the United Kingdom 1861-1957. Economica, 25(100), 283-299.

Rondina, F. (2018), Estimating unobservable inflation expectations in the New Keynesian Phillips Curve. Econometrics, 6, 1-20.

Samuelson, P.A., Solow, R. (1960), Analytical aspects of anti-inflation policy. American Economic Review, 50(2), 177-194.

Stock, J., Watson, M. (2008), Phillips Curve Inflation Forecasts. Vol. 53. NBER Working Papers, No.14322. p1-82.

Taghizadeh-Hesary, F., Rasoulinezhad, E., Yoshino, N. (2018), Volatility Linkages between Energy and Food Prices: Case of Selected Asian Countries. Vol. 829. Chiyoda City, Tokyo: Asian Development Bank Institute. p1-25.

Taylor, J. (1980), Aggregate Dynamics and Staggered Contracts. Vol. 88. Chicago, Illinois: University of Chicago Press. p1-23.